Polarization State Controlled Multistability of a Nonlinear Magneto-optic Cavity

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We investigate the theory of a nonlinear magneto-optic Fabry-Pérot interferometer, filled with an isotropic dielectric that possesses linear and nonlinear artificial gyrotropy. We show that the nonreciprocity leads to specific multistable transmission patterns and, in particular, to a polarization controlled multistability at constant input intensity. We also show that the reciprocity of the cavity can be restored effectively for certain parameter regimes. [S0031-9007(99)08486-0]

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The polarization state of the light reflects the vector nature of the electromagnetic field and introduces topological features with important implications regarding discrimination and robustness of certain electromagnetic interactions. This is particularly striking when nonreciprocity comes into play, and can have some far reaching conceptual repercussions in applications. This is, for instance, the case when unidirectional control or shielding of optical signal transfer is an issue, or in connection with storage and transfer of coherence, quantum optical or spin coherence in particular. In this respect magneto-optical interactions provide the most appropriate ground to study such aspects because of their gyrotropic character.

We address here such a case in confined geometry, namely, a magneto-optic Fabry-Pérot (FP) cavity, and in particular we show that this can exhibit a polarization state controlled multistable operation. The starting point is that the constitutive relation between the electric displacement vector $\mathbf{D}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{D}_\omega e^{-i\omega t}]$ and the electric field $\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{E}_\omega e^{-i\omega t}]$ can be written [1] in the form,

$$\mathbf{D}_\omega = \varepsilon_0(\boldsymbol{\varepsilon}: \mathbf{E}_\omega + i\mathbf{E}_\omega \times \mathbf{g}), \qquad (1)$$

where $\boldsymbol{\varepsilon}$ is the tensor of the electrical permittivity, and $\mathbf{g}$ the gyration vector, parallel to an externally applied magnetic field. Clearly the vector product in Eq. (1) lifts the degeneracy of left- and right-circular polarization states and introduces a nonreciprocal artificial gyrotropy. This aspect is strikingly manifested in normal reflection, and we anticipate this to have a distinct impact on the transmission characteristics of a magneto-optic FP cavity, because of the cumulative effect of multiple reflections and optical nonlinearities related to photoinduced modifications of $\boldsymbol{\varepsilon}$ and $\mathbf{g}$, namely, the optical Kerr effect and the photoinduced Faraday rotation [2], respectively. A constitutive relation of the form (1) leading to gyrotropy also holds [1] in a medium with natural optical activity (rotatory power) without placing it in a static magnetic field, but the reciprocity is preserved and the effects we discuss below cannot take place there.

The magneto-optic FP cavity consists of a dielectric placed between two partially reflecting plane surfaces, separated by a distance L, in the Faraday configuration with the optical fields propagating collinearly along the direction of the externally applied static magnetic field $\mathbf{H}_0 = H_0\mathbf{e}_z$. Keeping only lowest order optical and magneto-optical nonlinearities, in the infinite plane-wave limit, the field inside the cavity then obeys [3] the wave equation,

$$\frac{\partial^2 \mathbf{E}_\omega}{\partial z^2} + \frac{\omega^2}{c^2}\mathbf{E}_\omega = -\omega^2\mu_0 \sum_{j=1}^{4} \mathbf{P}_\omega^{(j)}, \qquad (2)$$

with $\mathbf{P}_\omega^{(1)} = \varepsilon_0 \boldsymbol{\chi}^{(ee)}: \mathbf{E}_\omega$ and $\mathbf{P}_\omega^{(3)} = \varepsilon_0 \boldsymbol{\chi}^{(eeee)}: \mathbf{E}_\omega \mathbf{E}_\omega \mathbf{E}_\omega^*$ being the linear and cubic (optical Kerr effect) [3] polarization densities, respectively; $\mathbf{P}_\omega^{(2)} = \varepsilon_0 \boldsymbol{\chi}^{(eem)}: \mathbf{E}_\omega \mathbf{H}_0$ and $\mathbf{P}_\omega^{(4)} = \varepsilon_0 \boldsymbol{\chi}^{(eeeem)}: \mathbf{E}_\omega \mathbf{E}_\omega \mathbf{E}_\omega^* \mathbf{H}_0$ are their modifications brought by the magnetic field, the linear and photoinduced Faraday effects, respectively [1,2]. We have introduced the nonlinear susceptibility formalism [3] and neglected the magneto-optical self-coupling to the weak and rapidly oscillating magnetic field of the light.

We introduce the circularly polarized basis $\mathbf{e}_\pm = (\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2}$, and by separating Eq. (2) into circularly polarized components $E_\pm = \mathbf{e}_\pm^* \cdot \mathbf{E}_\omega$ we obtain the system of nonlinear coupled differential equations,

$$\frac{\partial^2 E_\pm}{\partial z^2} + \frac{\omega^2}{c^2}(n^2 \pm \gamma + r_{1_\pm}|E_\pm|^2 + r_{2_\pm}|E_\mp|^2)E_\pm = 0, \qquad (3)$$

with $n^2 = 1 + \chi_{xx}^{(ee)}$, $\gamma = i\chi_{xyz}^{(eem)} H_0$, and where $r_{k_\pm} = p_k \pm q_k$, $k = 1, 2$, $p_{1,2} = (3/4)(\chi_{xxxx}^{(eeee)} \mp \chi_{xyyx}^{(eeee)})$, $q_{1,2} = i(3/4)(\chi_{xyyyz}^{(eeeem)} \mp \chi_{xxxyz}^{(eeeem)})H_0$, where we made use of intrinsic permutation symmetry [3] of the involved susceptibility tensors.

Resolving $E_\pm$ into their forward and backward traveling components, $\mathbf{E}_\omega = \mathbf{E}_\omega^f + \mathbf{E}_\omega^b$, $\mathbf{E}_\omega^f = \mathbf{e}_+ E_+^f e^{ik_0nz} + \mathbf{e}_- E_-^f e^{ik_0nz}$, $\mathbf{E}_\omega^b = \mathbf{e}_+^* E_+^b e^{-ik_0nz} + \mathbf{e}_-^* E_-^b e^{-ik_0nz}$, with $k_0 = \omega/c$, we can derive equations for the envelope

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functions $E^f_\pm$ and $E^b_\pm$, where a "+" in the subscript denotes left-circular polarization (LCP) and a "−" denotes right-circular polarization (RCP). We will from now on assume a lossless medium with real coefficients n, γ, and $r_{k_\pm}$, $k = 1, 2$. Applying the slowly varying envelope approximation [3], multiplying Eq. (3) by $e^{\pm ik_0 nz}$, and averaging over a few spatial periods then gives a system of four nonlinear coupled differential equations,

$$\frac{\partial E^f_\pm}{\partial z} = i\,\frac{k_0}{2n}\,[\pm\gamma + r_{1_\pm}(|E^f_\pm|^2 + 2|E^b_\mp|^2) + r_{2_\pm}(|E^f_\mp|^2 + |E^b_\pm|^2)]E^f_\pm\,, \qquad (4a)$$

$$\frac{\partial E^b_\pm}{\partial z} = -i\,\frac{k_0}{2n}\,[\mp\gamma + r_{1_\mp}(|E^b_\pm|^2 + 2|E^f_\mp|^2) + r_{2_\mp}(|E^b_\mp|^2 + |E^f_\pm|^2)]E^b_\pm\,, \qquad (4b)$$

where we neglected phase-mismatched terms. Equations (4) are easily integrated to give the general solution for the envelopes as

$$E^f_\pm = A^f_\pm e^{ik_0\eta^f_\pm z + i\psi^f_\pm}, \qquad E^b_\pm = A^b_\pm e^{-ik_0\eta^b_\pm z + i\psi^b_\pm}, \qquad (5)$$

with $A^{f,b}_\pm$ being positive constants of integration, $\psi^{f,b}_\pm$ the phases of a respective wave at the first reflecting surface of the cavity, at $z = 0$, and where

$$\eta^f_\pm = [\pm\gamma + r_{1_\pm}(A^{f\,2}_\pm + 2A^{b\,2}_\mp) + r_{2_\pm}(A^{f\,2}_\mp + A^{b\,2}_\pm)]/(2n)\,,$$

$$\eta^b_\pm = [\mp\gamma + r_{1_\mp}(A^{b\,2}_\pm + 2A^{f\,2}_\mp) + r_{2_\mp}(A^{b\,2}_\mp + A^{f\,2}_\pm)]/(2n)\,.$$

The boundary conditions of the cavity are

$$A^f_\pm e^{i\psi^f_\pm} = \tau^{(0)}_\pm E^I_\pm + \rho^{(0)}_\mp A^b_\mp e^{i\psi^b_\mp}, \qquad (6a)$$

$$A^b_\pm e^{-i(n+\eta^b_\pm)k_0L + i\psi^b_\pm} = \rho^{(1)}_\mp A^f_\mp e^{i(n+\eta^f_\mp)k_0L + i\psi^f_\mp}, \qquad (6b)$$

where $E^I_\pm$ are the incident complex fields taken immediately before the first reflecting surface. In Eqs. (6), $\tau^{(0)}_\pm = 2n_0/(n_0 + n_\pm)$, $\rho^{(0)}_\pm = (n_\mp - n_0)/(n_\mp + n_0)$, and $\rho^{(1)}_\pm = (n_\pm - n_1)/(n_\pm + n_1)$ are the complex amplitude transmission and reflection coefficients for LCP and RCP, with n_0 being the refractive index of the medium surrounding the cavity for $z < 0$, n_1 the refractive index for $L < z$, and $n^2_\pm = n^2 \pm \gamma$.

Using the fact that the electrical fields transmitted from the cavity are $E^T_\pm = \tau^{(1)}_\pm \mathbf{e}^*_\pm \cdot \mathbf{E}^f_\omega(z = L)$, with $\tau^{(1)}_\pm = 2n_\pm/(n_\pm + n_1)$, and introducing the new normalized and dimensionless variables $s^{I,T}_\pm = (k_0Lp_1/n) \times |E^{I,T}_\pm|^2$, Eqs. (5) and (6) can be reduced to

$$[1 + F_\pm \sin^2(\Gamma_\pm + C^{(1)}_\pm s^T_\pm + C^{(2)}_\mp s^T_\mp)]s^T_\pm = U_\pm s^I_\pm\,, \qquad (7)$$

where the constants $C^{(k)}_\pm$, $F_\pm$, and $U_\pm$ are defined as

$$C^{(1)}_\pm = \frac{3(1 + |\rho^{(1)}_\pm|^2)}{4|\tau^{(1)}_\pm|^2}[1 \mp \delta(1 - \nu_m)/(1 - \nu_e)]\,,$$

$$C^{(2)}_\pm = \frac{(1 + |\rho^{(1)}_\pm|^2)}{2|\tau^{(1)}_\pm|^2}\,\mu[1 \pm \delta(1 + \nu_m)/(1 + \nu_e)]\,,$$

$$F_\pm = \frac{4|\rho^{(0)}_\mp\rho^{(1)}_\pm|}{(1 - |\rho^{(0)}_\mp\rho^{(1)}_\pm|)^2}\,, \qquad U_\pm = \frac{|\tau^{(0)}_\pm\tau^{(1)}_\pm|^2}{(1 - |\rho^{(0)}_\mp\rho^{(1)}_\pm|)^2}\,,$$

$$\Gamma_\pm = [(2n \pm \gamma/n)k_0L + \arg\rho^{(0)}_\mp + \arg\rho^{(1)}_\pm]/2\,,$$

and where we defined the parameters

$$\nu_e = \chi^{(eeee)}_{xyyx}/\chi^{(eeee)}_{xxxx}\,, \qquad \delta = -iH_0\chi^{(eeeem)}_{xyyyz}/\chi^{(eeee)}_{xxxx}\,,$$

$$\nu_m = \chi^{(eeeem)}_{xxxyz}/\chi^{(eeeem)}_{xyyyz}\,, \qquad \mu = (1 + \nu_e)/(1 - \nu_e)\,.$$

In Eq. (7), $F_\pm$ have the role of the finesses of the cavity for pure LCP or RCP waves, and $\Gamma_\pm$ are half the total phase shifts experienced by pure LCP or RCP waves during one complete round-trip inside the cavity, excluding nonlinear effects.

When the incident light is either LCP or RCP, one can obtain exact solutions to Eq. (3). In particular, whenever the reflection coefficients are real, the exact solution becomes

$$[1 + F_\pm\,\mathrm{sn}^2(\nu_\pm k_0L, m_\pm)]s^T_\pm = U_\pm s^I_\pm\,,$$

where sn is a Jacobian elliptic function [4], with the parameters $\nu_\pm$ and $m_\pm$ given as

$$\nu^2_\pm = n^2_\pm + \left[1 + \frac{2|\rho^{(1)}_\pm|}{3(1 + |\rho^{(1)}_\pm|^2)}\right]\frac{2n}{k_0L}\,C^{(1)}_\pm s^T_\pm\,,$$

$$m_\pm = \frac{8|\rho^{(1)}_\pm|}{3(1 + |\rho^{(1)}_\pm|^2)}\,\frac{n}{\nu^2_\pm k_0L}\,C^{(1)}_\pm s^T_\pm\,.$$

The total solution for the transmitted light is conveniently expressed in terms of normalized and dimensionless Stokes parameters [5], $s_k = (k_0Lp_1/n)S_k$, $k = 0, 1, 2, 3$, with

$$S_0 = |E^T_+|^2 + |E^T_-|^2, \qquad S_1 = 2\,\mathrm{Re}[E^{T*}_+E^T_-]\,,$$

$$S_3 = |E^T_+|^2 - |E^T_-|^2, \qquad S_2 = 2\,\mathrm{Im}[E^{T*}_+E^T_-]\,,$$

and the incident light by the set $w_k = (k_0Lp_1/n)W_k$, $k = 0, 1, 2, 3$, with W_k defined similarly to S_k, with T replaced by I. Using this transformation, Eq. (7) becomes

$$(1 + F_\pm \sin^2\xi_\pm)(s_0 \pm s_3) = U_\pm(w_0 \pm w_3)\,, \qquad (8)$$

with $\xi_\pm = \xi_\pm(s_0, s_3)$ given as

$$\xi_\pm = \Gamma \pm \Delta\Gamma + (C^{(1)}_\pm + C^{(2)}_\mp)s_0/2 \pm (C^{(1)}_\pm - C^{(2)}_\mp)s_3/2\,,$$

where we defined the detuning angle $\Gamma = (\Gamma_+ + \Gamma_-)/2$ (mod π) and differential detuning angle $\Delta\Gamma = (\Gamma_+ - \Gamma_-)/2$ (mod π). The effective impact of nonreciprocity is manifested whenever Eq. (8) is noninvariant under the transformation $E^I_\pm \to E^{I*}_\mp$, $E^T_\pm \to E^{T*}_\mp$, i.e., $(w_3, s_3) \to -(w_3, s_3)$. The remaining Stokes parameters, s_1 and s_2, are obtained from Eqs. (5) and (6) as

$$s_1 = (s_0^2 - s_3^2)^{1/2} \cos(\varphi),$$
$$s_2 = (s_0^2 - s_3^2)^{1/2} \sin(\varphi), \tag{9}$$

with $\varphi = \varphi(s_0, s_3)$ being twice the angle between the x axis and the main axis of the polarization ellipse of the transmitted light,

$$\varphi = \varphi_0 + \varphi_1 + \varphi_2 + \varphi_3,$$
$$\varphi_0 = \arg E^I_- - \arg E^I_+,$$
$$\varphi_1 = \arg \tau^{(0)}_- + \arg \tau^{(1)}_- - \arg \tau^{(0)}_+ - \arg \tau^{(1)}_+,$$
$$\varphi_2 = -k_0\gamma L/n - (D_+ + D_-)s_0 - (D_+ - D_-)s_3,$$
$$\varphi_3 = \arctan\left(\frac{|\rho^{(0)}_+ \rho^{(1)}_-| \sin 2\xi_-}{1 - |\rho^{(0)}_+ \rho^{(1)}_-| \cos 2\xi_-}\right) - \arctan\left(\frac{|\rho^{(0)}_- \rho^{(1)}_+| \sin 2\xi_+}{1 - |\rho^{(0)}_- \rho^{(1)}_+| \cos 2\xi_+}\right),$$
$$D_\pm = \frac{(1 + 2|\rho^{(1)}_\pm|^2)}{3(1 + |\rho^{(1)}_\pm|^2)} C^{(1)}_\pm - \frac{1}{2} C^{(2)}_\pm.$$

The contribution φ_3 follows directly from the nonreciprocity of the artificially induced gyrotropy, as opposite to the reciprocal natural gyrotropy (optical activity), where the rotation of the polarization ellipse of the forward traveling field is compensated for on the way back [1,6]. Relations (8) and (9) completely describe the polarization pattern of the transmitted beam for an arbitrary polarization of the incident beam. From this complex pattern we single out the case of polarization state controlled multistable transmission.

The following discussion is significantly simplified under the approximation of equal reflectivities for LCP and RCP of both surfaces, $\rho^{(k)}_\pm \approx \rho$, $k = 0, 1$.

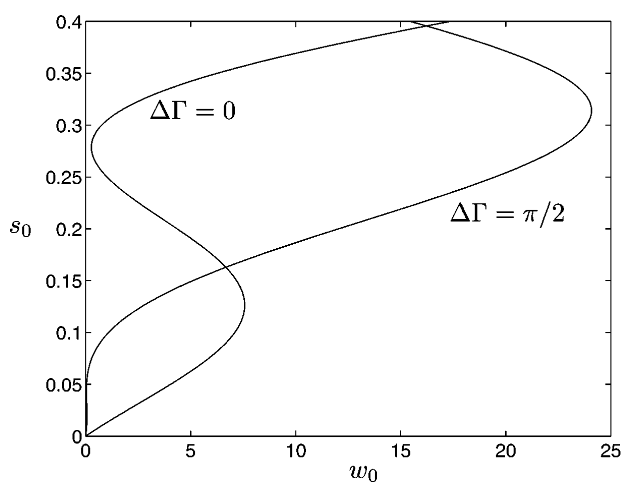

FIG. 1. Normalized transmitted intensity s_0 vs normalized input intensity w_0. Used parameter values are $R = 0.8$, $\Gamma = 0.4\pi$, $\nu_e = 0.2$, and $\delta = 0$.

First we analyze the influence of the linear gyrotropy in the case with negligible nonlinear gyrotropy, namely, $\delta = 0$. We note that, for differential detuning angles $\Delta\Gamma = 0, \pi/2$, the effective impact of the nonreciprocity on the transmitted light disappears, and for $\nu_e = 0.2$ the transmitted intensity (ellipticity) becomes independent of the input ellipticity (intensity). In Fig. 1 we show the normalized transmitted intensity s_0, obtained from Eq. (8), vs normalized input intensity w_0 for these two extreme cases, for $R = |\rho|^2 = 0.8$, $\Gamma = 0.4\pi$, and an arbitrary polarization state; a multistable behavior is obtained as in the ordinary scalar FP cavity [7,8]. As the differential detuning angle is varied, for example, starting with $\Delta\Gamma = 0$ and increasing the externally applied magnetic field H_0, for $\Delta\Gamma = 0.2\pi$ we obtain the solution shown in Fig. 2. Figure 2(a) clearly shows the asymmetrical impact of the

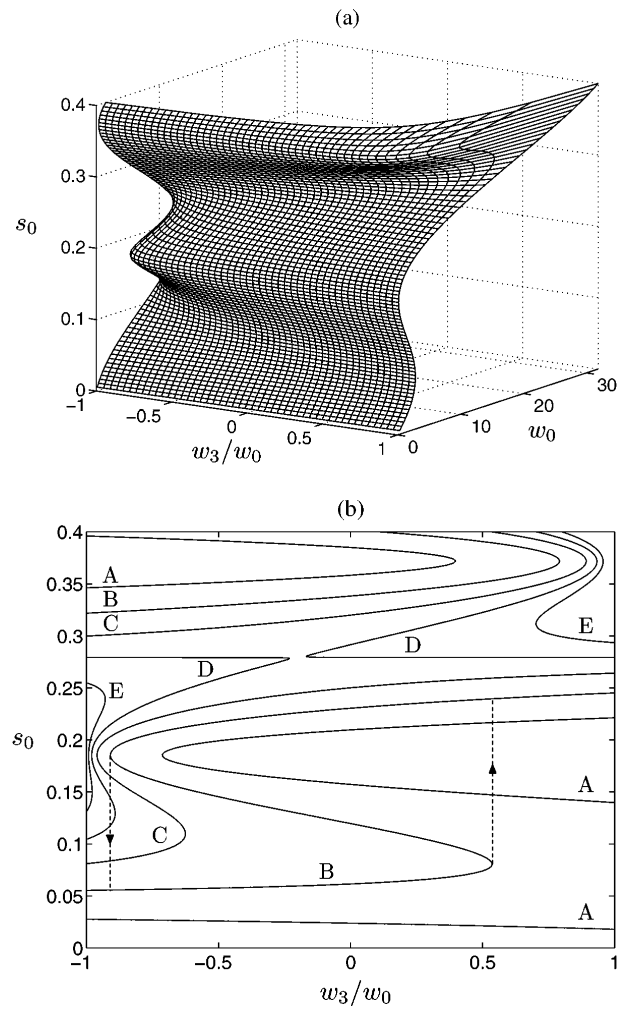

FIG. 2. (a) Normalized transmitted intensity s_0 vs normalized input intensity w_0 and normalized input ellipticity w_3/w_0. (b) Normalized transmitted intensity vs normalized input ellipticity, taken for (*A*) $\omega_0 = 2.0$, (*B*) $\omega_0 = 3.2$, (*C*) $\omega_0 = 5.6$, (*D*) $\omega_0 = 8.0$, and (*E*) $\omega_0 = 10.6$. One optical hysteresis loop is indicated by the dashed arrows. Used parameter values are $R = 0.8$, $\Gamma = 0.4\pi$, $\Delta\Gamma = 0.2\pi$, $\nu_e = 0.2$, and $\delta = 0$. Notice the asymmetrical impact of nonreciprocity.

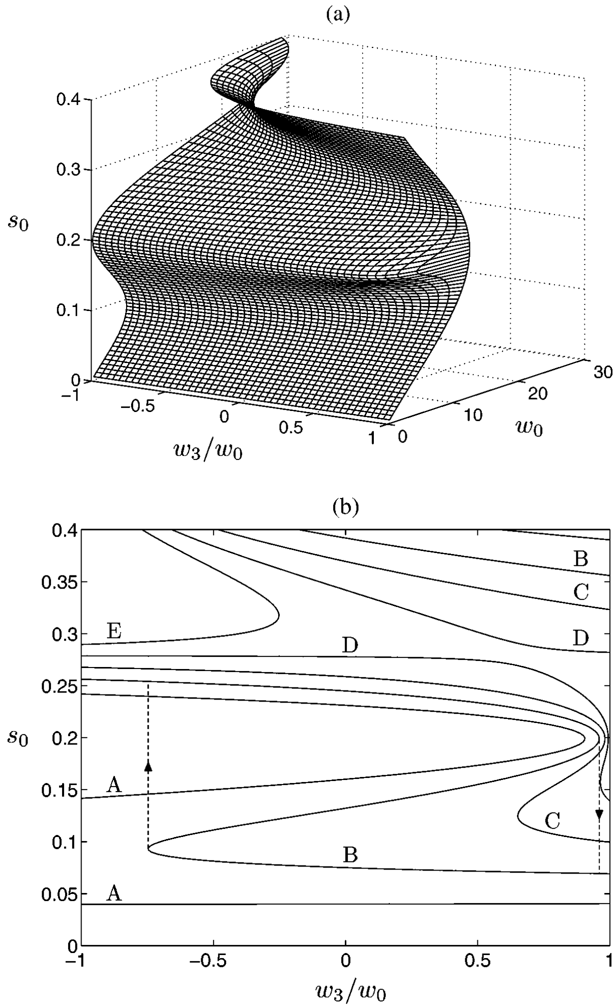

FIG. 3. (a) Normalized transmitted intensity s_0 vs normalized input intensity w_0 and input ellipticity w_3/w_0. (b) Normalized transmitted intensity vs input ellipticity, taken for (*A*) $\omega_0 = 3.2$, (*B*) $\omega_0 = 5.6$, (*C*) $\omega_0 = 8.0$, (*D*) $\omega_0 = 10.6$, and (*E*) $\omega_0 = 13.4$. One optical hysteresis loop is indicated by the dashed arrows. Used parameter values are $R = 0.8$, $\Gamma = 0.4\pi$, $\Delta\Gamma = 0$, $\nu_e = \nu_m = 0.2$, and $\delta = 0.4$.

nonreciprocity on the transmitted light. The striking result is the appearance of hysteresis loops at constant input intensity, as shown in Fig. 2(b). Further increase of the magnetic field causes the FP interferometer at $\Delta\Gamma = \pi/2$ to return to a state where the nonreciprocity is effectively cancelled, with the transmitted intensity (ellipticity) becoming independent of the input ellipticity (intensity), as shown in Fig. 1.

We now analyze the influence of nonlinear magneto-optical interaction, and in order to contrast it with the previous case we choose $\Delta\Gamma = 0$, and all other parameter values same as before; in addition, we choose $\nu_m = 0.2$. As the nonlinear magneto-optical interaction increases from $\delta = 0$ the nonreciprocity sets in, and the transmitted intensity (ellipticity) again is no longer independent on the input ellipticity (intensity); the interdependence becomes more complex with increasing δ. In Fig. 3 we show a case for $\delta = 0.4$. As shown in Fig. 3(b) hysteresis loops appear for constant input intensity as in the previous case with $\delta = 0$ and $\Delta\Gamma \neq 0, \pi/2$. In fact, a careful analysis shows a whole series of regions where the behavior becomes effectively reciprocal because of exact cancellation of linear and photoinduced terms of Eqs. (8) and (9).

Several systems can be considered for experimental investigations. In the transparency region we considered here the nonlinear coefficients are weak, but close to resonances they can be appreciable and the predicted behavior comfortably observable. Some provisions must be made in the previous theory to include absorption losses. One interesting case is that of semimagnetic semiconductors where giant linear and photoinduced Faraday rotations have been evidenced [9] both in bulk and in multiple quantum wells with moderate magnetic field and beam intensity. Another case [10] is that of an atomic gas, for instance, sodium or cesium, where again close to an atomic resonance giant Faraday rotations have been measured [11,12] and similarly for rare earth doped crystals or glasses.

In conclusion, we have derived the transmission characteristics of a nonlinear magneto-optic cavity. The solutions show a rich behavior as a consequence of the interplay between nonlinearity and gyrotropy, and one of the impacts of the artificially introduced gyrotropy is the appearance of a polarization state controlled multistability at constant input light intensity.

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