9.3.3

$$J_\nu(\nu\sec\beta)=\sqrt{2/(\pi\nu\tan\beta)}\{\cos(\nu\tan\beta-\nu\beta-\tfrac{1}{4}\pi)+O(\nu^{-1})\}\qquad(0<\beta<\tfrac{1}{2}\pi)$$

$$Y_\nu(\nu\sec\beta)=\sqrt{2/(\pi\nu\tan\beta)}\{\sin(\nu\tan\beta-\nu\beta-\tfrac{1}{4}\pi)+O(\nu^{-1})\}\qquad(0<\beta<\tfrac{1}{2}\pi)$$

9.3.4

$$J_\nu(\nu+z\nu^{1/3})=2^{1/3}\nu^{-1/3}\,\mathrm{Ai}(-2^{1/3}z)+O(\nu^{-1})$$

$$Y_\nu(\nu+z\nu^{1/3})=-2^{1/3}\nu^{-1/3}\,\mathrm{Bi}(-2^{1/3}z)+O(\nu^{-1})$$

9.3.5

$$J_\nu(\nu)\sim\frac{2^{1/3}}{3^{2/3}\Gamma(\frac{2}{3})}\frac{1}{\nu^{1/3}}$$

$$Y_\nu(\nu)\sim-\frac{2^{1/3}}{3^{1/6}\Gamma(\frac{2}{3})}\frac{1}{\nu^{1/3}}$$

9.3.6

$$J_\nu(\nu z)=\left(\frac{4\zeta}{1-z^2}\right)^{1/4}\left\{\frac{\mathrm{Ai}(\nu^{2/3}\zeta)}{\nu^{1/3}}+\frac{\exp(-\frac{2}{3}\nu\zeta^{3/2})}{1+\nu^{1/6}|\zeta|^{1/4}}\,O\left(\frac{1}{\nu^{4/3}}\right)\right\}\qquad(|\arg z|<\pi)$$

$$Y_\nu(\nu z)=-\left(\frac{4\zeta}{1-z^2}\right)^{1/4}\left\{\frac{\mathrm{Bi}(\nu^{2/3}\zeta)}{\nu^{1/3}}+\frac{\exp|\mathscr{R}(\frac{2}{3}\nu\zeta^{3/2})|}{1+\nu^{1/6}|\zeta|^{1/4}}\,O\left(\frac{1}{\nu^{4/3}}\right)\right\}\qquad(|\arg z|<\pi)$$

In the last two equations ζ is given by **9.3.38** and **9.3.39** below.

Debye's Asymptotic Expansions

(i) If α is fixed and positive and ν is large and positive

9.3.7

$$J_\nu(\nu\,\mathrm{sech}\,\alpha)\sim\frac{e^{\nu(\tanh\alpha-\alpha)}}{\sqrt{2\pi\nu\tanh\alpha}}\left\{1+\sum_{k=1}^{\infty}\frac{u_k(\coth\alpha)}{\nu^k}\right\}$$

9.3.8

$$Y_\nu(\nu\,\mathrm{sech}\,\alpha)\sim-\frac{e^{\nu(\alpha-\tanh\alpha)}}{\sqrt{\frac{1}{2}\pi\nu\tanh\alpha}}\left\{1+\sum_{k=1}^{\infty}(-)^k\frac{u_k(\coth\alpha)}{\nu^k}\right\}$$

where

9.3.9

$$u_0(t)=1$$
$$u_1(t)=(3t-5t^3)/24$$
$$u_2(t)=(81t^2-462t^4+385t^6)/1152$$
$$u_3(t)=(30375t^3-3\,69603t^5+7\,65765t^7-4\,25425t^9)/4\,14720$$
$$u_4(t)=(44\,65125t^4-941\,21676t^6+3499\,22430t^8-4461\,85740t^{10}+1859\,10725t^{12})/398\,13120$$

For $u_5(t)$ and $u_6(t)$ see [9.4] or [9.21].

9.3.10

$$u_{k+1}(t)=\tfrac{1}{2}t^2(1-t^2)u_k'(t)+\frac{1}{8}\int_0^t(1-5t^2)u_k(t)dt\qquad(k=0,1,\ldots)$$

Also

9.3.11

$$J_\nu'(\nu\,\mathrm{sech}\,\alpha)\sim\sqrt{\frac{\sinh 2\alpha}{4\pi\nu}}\,e^{\nu(\tanh\alpha-\alpha)}\left\{1+\sum_{k=1}^{\infty}\frac{v_k(\coth\alpha)}{\nu^k}\right\}$$

9.3.12

$$Y_\nu'(\nu\,\mathrm{sech}\,\alpha)\sim\sqrt{\frac{\sinh 2\alpha}{\pi\nu}}\,e^{\nu(\alpha-\tanh\alpha)}\left\{1+\sum_{k=1}^{\infty}(-)^k\frac{v_k(\coth\alpha)}{\nu^k}\right\}$$

where

9.3.13

$$v_0(t)=1$$
$$v_1(t)=(-9t+7t^3)/24$$
$$v_2(t)=(-135t^2+594t^4-455t^6)/1152$$
$$v_3(t)=(-42525t^3+4\,51737t^5-8\,83575t^7+4\,75475t^9)/4\,14720$$

9.3.14

$$v_k(t)=u_k(t)+t(t^2-1)\{\tfrac{1}{2}u_{k-1}(t)+tu_{k-1}'(t)\}\qquad(k=1,2,\ldots)$$

(ii) If β is fixed, $0<\beta<\frac{1}{2}\pi$ and ν is large and positive

9.3.15

$$J_\nu(\nu\sec\beta)=\sqrt{2/(\pi\nu\tan\beta)}\{L(\nu,\beta)\cos\Psi+M(\nu,\beta)\sin\Psi\}$$

9.3.16

$$Y_\nu(\nu\sec\beta)=\sqrt{2/(\pi\nu\tan\beta)}\{L(\nu,\beta)\sin\Psi-M(\nu,\beta)\cos\Psi\}$$

where $\Psi=\nu(\tan\beta-\beta)-\frac{1}{4}\pi$

9.3.17

$$L(\nu,\beta)\sim\sum_{k=0}^{\infty}\frac{u_{2k}(i\cot\beta)}{\nu^{2k}}=1-\frac{81\cot^2\beta+462\cot^4\beta+385\cot^6\beta}{1152\nu^2}+\ldots$$