Derivatives With Respect to Order

10.1.41

$$\left[\frac{\partial}{\partial\nu}j_\nu(x)\right]_{\nu=0}=(\tfrac{1}{2}\pi/x)\{\mathrm{Ci}(2x)\sin x-\mathrm{Si}(2x)\cos x\}$$

10.1.42

$$\left[\frac{\partial}{\partial\nu}j_\nu(x)\right]_{\nu=-1}=(\tfrac{1}{2}\pi/x)\{\mathrm{Ci}(2x)\cos x+\mathrm{Si}(2x)\sin x\}$$

10.1.43

$$\left[\frac{\partial}{\partial\nu}y_\nu(x)\right]_{\nu=0}=(\tfrac{1}{2}\pi/x)\{\mathrm{Ci}(2x)\cos x+[\mathrm{Si}(2x)-\pi]\sin x\}$$

10.1.44

$$\left[\frac{\partial}{\partial\nu}y_\nu(x)\right]_{\nu=-1}=$$
$$(\tfrac{1}{2}\pi/x)\{\mathrm{Ci}(2x)\sin x-[\mathrm{Si}(2x)-\pi]\cos x\}$$

Addition Theorems and Degenerate Forms

r, ρ, θ, λ arbitrary complex; $R=\sqrt{(r^2+\rho^2-2r\rho\cos\theta)}$

10.1.45 $$\frac{\sin\lambda R}{\lambda R}=\sum_0^\infty(2n+1)j_n(\lambda r)j_n(\lambda\rho)P_n(\cos\theta)$$

***10.1.46** $$-\frac{\cos\lambda R}{\lambda R}=\sum_0^\infty(2n+1)j_n(\lambda r)y_n(\lambda\rho)P_n(\cos\theta)\qquad |re^{\pm i\theta}|<|\rho|$$

10.1.47 $$e^{iz\cos\theta}=\sum_0^\infty(2n+1)e^{\frac{1}{2}n\pi i}j_n(z)P_n(\cos\theta)$$

10.1.48

$$J_0(z\sin\theta)=\sum_0^\infty(4n+1)\frac{(2n)!}{2^{2n}(n!)^2}j_{2n}(z)P_{2n}(\cos\theta)$$

Duplication Formula

10.1.49

$$j_n(2z)=$$

* $$-n!z^{n+1}\sum_0^n\frac{2n-2k+1}{k!(2n-k+1)!}j_{n-k}(z)y_{n-k}(z)$$

Some Infinite Series Involving $j_n^2(z)$

10.1.50 $$\sum_0^\infty(2n+1)j_n^2(z)=1$$

10.1.51 $$\sum_0^\infty(-1)^n(2n+1)j_n^2(z)=\frac{\sin 2z}{2z}$$

10.1.52 $$\sum_0^\infty j_n^2(z)=\frac{\mathrm{Si}(2z)}{2z}$$

*See page II.

Fresnel Integrals

10.1.53

$$C(\sqrt{2x/\pi})=\frac{1}{2}\int_0^x J_{-\frac{1}{2}}(t)dt$$
$$=\sqrt{2}[\cos\tfrac{1}{2}x\sum_0^\infty(-1)^nJ_{2n+\frac{1}{2}}(\tfrac{1}{2}x)$$
$$+\sin\tfrac{1}{2}x\sum_0^\infty(-1)^nJ_{2n+3/2}(\tfrac{1}{2}x)]$$

10.1.54

$$S(\sqrt{2x/\pi})=\frac{1}{2}\int_0^x J_{\frac{1}{2}}(t)dt$$
$$=\sqrt{2}[\sin\tfrac{1}{2}x\sum_0^\infty(-1)^nJ_{2n+\frac{1}{2}}(\tfrac{1}{2}x)$$
$$-\cos\tfrac{1}{2}x\sum_0^\infty(-1)^nJ_{2n+3/2}(\tfrac{1}{2}x)].$$

(See also **11.1.1, 11.1.2.**)

Zeros and Their Asymptotic Expansions

The zeros of $j_n(x)$ and $y_n(x)$ are the same as the zeros of $J_{n+\frac{1}{2}}(x)$ and $Y_{n+\frac{1}{2}}(x)$ and the formulas for $j_{\nu,s}$ and $y_{\nu,s}$ given in **9.5** are applicable with $\nu=n+\frac{1}{2}$. There are, however, no simple relations connecting the zeros of the derivatives. Accordingly, we now give formulas for $a'_{n,s}$, $b'_{n,s}$, the s-th positive zero of $j'_n(z)$, $y'_n(z)$, respectively; $z=0$ is counted as the first zero of $j'_0(z)$.

(Tables of $a'_{n,s}$, $b'_{n,s}$, $j_n(a'_{n,s})$, $y_n(b'_{n,s})$ are given in [10.31].)

Elementary Relations

$$f_n(z)=j_n(z)\cos\pi t+y_n(z)\sin\pi t$$

(t a real parameter, $0\le t\le 1$)

If τ_n is a zero of $f'_n(z)$ then

10.1.55 $$f_n(\tau_n)=[\tau_n/(n+1)]f_{n-1}(\tau_n)$$
(See **10.1.21.**)

10.1.56 $$=(\tau_n/n)f_{n+1}(\tau_n)$$
(See **10.1.22.**)

10.1.57 $$=\left\{\frac{1}{\pi}[\tau_n^2-n(n+1)]\frac{d\tau_n}{d\tau}\right\}^{-\frac{1}{2}}$$