Representations in Terms of Bessel Functions

$$\zeta=\tfrac{2}{3}z^{3/2}$$

10.4.14

$$\mathrm{Ai}\,(z)=\tfrac{1}{3}\sqrt{z}[I_{-1/3}(\zeta)-I_{1/3}(\zeta)]=\pi^{-1}\sqrt{z/3}K_{1/3}(\zeta)$$

10.4.15

$$\mathrm{Ai}\,(-z)=\tfrac{1}{3}\sqrt{z}[J_{1/3}(\zeta)+J_{-1/3}(\zeta)]$$
$$=\tfrac{1}{2}\sqrt{z/3}[e^{\pi i/6}H^{(1)}_{1/3}(\zeta)+e^{-\pi i/6}H^{(2)}_{1/3}(\zeta)]$$

10.4.16

* $$-\mathrm{Ai}'(z)=\tfrac{1}{3}z[I_{-2/3}(\zeta)-I_{2/3}(\zeta)]=\pi^{-1}(z/\sqrt{3})K_{2/3}(\zeta)$$

10.4.17

$$\mathrm{Ai}'\,(-z)=-\tfrac{1}{3}z[J_{-2/3}(\zeta)-J_{2/3}(\zeta)]$$
$$=\tfrac{1}{2}(z/\sqrt{3})[e^{-\pi i/6}H^{(1)}_{2/3}(\zeta)+e^{\pi i/6}H^{(2)}_{2/3}(\zeta)]$$

10.4.18 $$\mathrm{Bi}\,(z)=\sqrt{z/3}[I_{-1/3}(\zeta)+I_{1/3}(\zeta)]$$

10.4.19

$$\mathrm{Bi}\,(-z)=\sqrt{z/3}[J_{-1/3}(\zeta)-J_{1/3}(\zeta)]$$
$$=\tfrac{1}{2}i\sqrt{z/3}[e^{\pi i/6}H^{(1)}_{1/3}(\zeta)-e^{-\pi i/6}H^{(2)}_{1/3}(\zeta)]$$

10.4.20 $$\mathrm{Bi}'\,(z)=(z/\sqrt{3})[I_{-2/3}(\zeta)+I_{2/3}(\zeta)]$$

10.4.21

$$\mathrm{Bi}'\,(-z)=(z/\sqrt{3})[J_{-2/3}(\zeta)+J_{2/3}(\zeta)]$$
$$=\tfrac{1}{2}i(z/\sqrt{3})[e^{-\pi i/6}H^{(1)}_{2/3}(\zeta)-e^{\pi i/6}H^{(2)}_{2/3}(\zeta)]$$

Representations of Bessel Functions in Terms of Airy Functions

$$z=\left(\frac{3}{2}\zeta\right)^{2/3}$$

10.4.22 $$J_{\pm 1/3}(\zeta)=\tfrac{1}{2}\sqrt{3/z}[\sqrt{3}\,\mathrm{Ai}\,(-z)\mp\mathrm{Bi}\,(-z)]$$

***10.4.23** $$H^{(1)}_{\pm 1/3}(\zeta)=e^{\mp\pi i/6}\sqrt{3/z}[\mathrm{Ai}\,(-z)-i\,\mathrm{Bi}\,(-z)]$$

10.4.24 $$H^{(2)}_{\pm 1/3}(\zeta)=e^{\pm\pi i/6}\sqrt{3/z}[\mathrm{Ai}\,(-z)+i\,\mathrm{Bi}\,(-z)]$$

10.4.25 $$I_{\pm 1/3}(\zeta)=\tfrac{1}{2}\sqrt{3/z}[\mp\sqrt{3}\,\mathrm{Ai}\,(z)+\mathrm{Bi}\,(z)]$$

10.4.26 $$K_{\pm 1/3}(\zeta)=\pi\sqrt{3/z}\,\mathrm{Ai}\,(z)$$

10.4.27 $$J_{\pm 2/3}(\zeta)=(\sqrt{3}/2z)[\pm\sqrt{3}\,\mathrm{Ai}'\,(-z)+\mathrm{Bi}'\,(-z)]$$

10.4.28

$$H^{(1)}_{2/3}(\zeta)=e^{-2\pi i/3}H^{(1)}_{-2/3}(\zeta)$$
$$=e^{\pi i/6}(\sqrt{3}/z)[\mathrm{Ai}'\,(-z)-i\,\mathrm{Bi}'\,(-z)]$$

10.4.29

$$H^{(2)}_{2/3}(\zeta)=e^{2\pi i/3}H^{(2)}_{-2/3}(\zeta)$$
$$=e^{-\pi i/6}(\sqrt{3}/z)[\mathrm{Ai}'\,(-z)+i\,\mathrm{Bi}'\,(-z)]$$

10.4.30 $$I_{\pm 2/3}(\zeta)=(\sqrt{3}/2z)[\pm\sqrt{3}\,\mathrm{Ai}'\,(z)+\mathrm{Bi}'\,(z)]$$

10.4.31 $$K_{\pm 2/3}(\zeta)=-\pi(\sqrt{3}/z)\,\mathrm{Ai}'\,(z)$$

Integral Representations

10.4.32

$$(3a)^{-1/3}\pi\,\mathrm{Ai}\,[\pm(3a)^{-1/3}x]=\int_0^\infty \cos\,(at^3\pm xt)dt$$

10.4.33

$$(3a)^{-1/3}\pi\,\mathrm{Bi}\,[\pm(3a)^{-1/3}x]$$
$$=\int_0^\infty [\exp\,(-at^3\pm xt)+\sin\,(at^3\pm xt)]dt$$

The Integrals $\int_0^z \mathrm{Ai}\,(\pm t)dt$, $\int_0^z \mathrm{Bi}\,(\pm t)dt$

$$\zeta=\tfrac{2}{3}z^{3/2}$$

10.4.34 $$\int_0^z \mathrm{Ai}\,(t)dt=\frac{1}{3}\int_0^\zeta [I_{-1/3}(t)-I_{1/3}(t)]dt$$

10.4.35 $$\int_0^z \mathrm{Ai}\,(-t)dt=\frac{1}{3}\int_0^\zeta [J_{-1/3}(t)+J_{1/3}(t)]dt$$

10.4.36 $$\int_0^z \mathrm{Bi}\,(t)dt=\frac{1}{\sqrt{3}}\int_0^\zeta [I_{-1/3}(t)+I_{1/3}(t)]dt$$

10.4.37 $$\int_0^z \mathrm{Bi}\,(-t)dt=\frac{1}{\sqrt{3}}\int_0^\zeta [J_{-1/3}(t)-J_{1/3}(t)]dt$$

Ascending Series for $\int_0^z \mathrm{Ai}\,(\pm t)dt$, $\int_0^z \mathrm{Bi}\,(\pm t)dt$

10.4.38 $$\int_0^z \mathrm{Ai}\,(t)dt=c_1F(z)-c_2G(z)$$

(See **10.4.2.**)

10.4.39 $$\int_0^z \mathrm{Ai}\,(-t)dt=-c_1F(-z)+c_2G(-z)$$

10.4.40 $$\int_0^z \mathrm{Bi}\,(t)dt=\sqrt{3}[c_1F(z)+c_2G(z)]$$

(See **10.4.3.**)

10.4.41

$$\int_0^z \mathrm{Bi}\,(-t)dt=-\sqrt{3}[c_1F(-z)+c_2G(-z)]$$

$$F(z)=z+\frac{1}{4!}z^4+\frac{1\cdot 4}{7!}z^7+\frac{1\cdot 4\cdot 7}{10!}z^{10}+\ldots$$
$$=\sum_0^\infty 3^k\left(\frac{1}{3}\right)_k\frac{z^{3k+1}}{(3k+1)!}$$

$$G(z)=\frac{1}{2!}z^2+\frac{2}{5!}z^5+\frac{2\cdot 5}{8!}z^8+\frac{2\cdot 5\cdot 8}{11!}z^{11}+\ldots$$
$$=\sum_0^\infty 3^k\left(\frac{2}{3}\right)_k\frac{z^{3k+2}}{(3k+2)!}$$

The constants c_1, c_2 are given in **10.4.4, 10.4.5.**

*See page II.