**10.4.84** $\int_0^x \text{Bi}(t)dt \sim \pi^{-1/2}x^{-3/4}\exp\left(\frac{2}{3}x^{3/2}\right)$

**10.4.85** $\int_0^x \text{Bi}(-t)dt \sim \pi^{-1/2}x^{-3/4}\sin\left(\frac{2}{3}x^{3/2}+\frac{\pi}{4}\right)$

**Asymptotic Forms of Gi ($\pm x$), Gi′ ($\pm x$), Hi ($\pm x$), Hi′ ($\pm x$) for Large $x$**

**10.4.86** $\text{Gi}(x) \sim \pi^{-1}x^{-1}$

**10.4.87** $\text{Gi}(-x) \sim \pi^{-1/2}x^{-1/4}\cos\left(\frac{2}{3}x^{3/2}+\frac{\pi}{4}\right)$

**10.4.88** $\text{Gi}'(x) \sim \frac{7}{96}\pi^{-1}x^{-2}$

**10.4.89** $\text{Gi}'(-x) \sim \pi^{-1/2}x^{1/4}\sin\left(\frac{2}{3}x^{3/2}+\frac{\pi}{4}\right)$

**10.4.90** $\text{Hi}(x) \sim \pi^{-1/2}x^{-1/4}\exp\left(\tfrac{2}{3}x^{3/2}\right)$

**10.4.91** $\text{Hi}(-x) \sim \pi^{-1}x^{-1}$

**10.4.92** $\text{Hi}'(x) \sim \pi^{-1/2}x^{1/4}\exp\left(\tfrac{2}{3}x^{3/2}\right)$

**10.4.93** $\text{Hi}'(-x) \sim -\frac{3}{2}\pi^{-1}x^{-2}$

**Zeros and Their Asymptotic Expansions**

Ai ($z$), Ai′ ($z$) have zeros on the negative real axis only. Bi ($z$), Bi′ ($z$) have zeros on the negative real axis and in the sector $\frac{1}{3}\pi < |\arg z| < \frac{1}{2}\pi$. $a_s$, $a'_s$; $b_s$, $b'_s$ $s$-th (real) negative zero of Ai ($z$), Ai′ ($z$); Bi ($z$), Bi′ ($z$), respectively. $\beta_s$, $\beta'_s$; $\bar{\beta}_s$, $\bar{\beta}'_s$ $s$-th complex zero of Bi ($z$), Bi′ ($z$) in the sectors $\frac{1}{3}\pi < \arg z < \frac{1}{2}\pi$, $-\frac{1}{2}\pi < \arg z < -\frac{1}{3}\pi$, respectively.

**10.4.94** $a_s = -f[3\pi(4s-1)/8]$

**10.4.95** $a'_s = -g[3\pi(4s-3)/8]$

**10.4.96** $\text{Ai}'(a_s) = (-1)^{s-1}f_1[3\pi(4s-1)/8]$

**10.4.97** $\text{Ai}(a'_s) = (-1)^{s-1}g_1[3\pi(4s-3)/8]$

**10.4.98** $b_s = -f[3\pi(4s-3)/8]$

**10.4.99** $b'_s = -g[3\pi(4s-1)/8]$

**10.4.100** $\text{Bi}'(b_s) = (-1)^{s-1}f_1[3\pi(4s-3)/8]$

**10.4.101** $\text{Bi}(b'_s) = (-1)^{s}g_1[3\pi(4s-1)/8]$

**10.4.102** $\beta_s = e^{\pi i/3}f\left[\frac{3\pi}{8}(4s-1)+\frac{3i}{4}\ln 2\right]$

**10.4.103** $\beta'_s = e^{\pi i/3}g\left[\frac{3\pi}{8}(4s-3)+\frac{3i}{4}\ln 2\right]$

*See page II.

**10.4.104**

$$\text{Bi}'(\beta_s) = (-1)^s\sqrt{2}e^{-\pi i/6}f_1\left[\frac{3\pi}{8}(4s-1)+\frac{3i}{4}\ln 2\right]$$

**10.4.105**

$$\text{Bi}(\beta'_s) = (-1)^{s-1}\sqrt{2}e^{\pi i/6}g_1\left[\frac{3\pi}{8}(4s-3)+\frac{3i}{4}\ln 2\right]$$

$|z|$ sufficiently large

$$f(z) \sim z^{2/3}\left(1+\frac{5}{48}z^{-2}-\frac{5}{36}z^{-4}+\frac{77125}{82944}z^{-6} - \frac{1080\,56875}{69\,67296}z^{-8} + \frac{16\,23755\,96875}{3344\,30208}z^{-10} - \dots\right)$$

$$g(z) \sim z^{2/3}\left(1-\frac{7}{48}z^{-2}+\frac{35}{288}z^{-4}-\frac{181223}{207360}z^{-6} + \frac{186\,83371}{12\,44160}z^{-8} - \frac{9\,11458\,84361}{1911\,02976}z^{-10} + \dots\right)$$

$$f_1(z) \sim \pi^{-1/2}z^{1/6}\left(1+\frac{5}{48}z^{-2}-\frac{1525}{4608}z^{-4} + \frac{23\,97875}{6\,63552}z^{-6} - \dots\right)$$

$$g_1(z) \sim \pi^{-1/2}z^{-1/6}\left(1-\frac{7}{96}z^{-2}+\frac{1673}{6144}z^{-4} - \frac{843\,94709}{265\,42080}z^{-6} + \dots\right) \quad *$$

**Formal and Asymptotic Solutions of Ordinary Differential Equations of Second Order With Turning Points**

An equation

**10.4.106** $W''+a(z,\lambda)W'+b(z,\lambda)W=0$

in which $\lambda$ is a real or complex parameter and, for fixed $\lambda$, $a(z,\lambda)$ is analytic in $z$ and $b(z,\lambda)$ is continuous in $z$ in some region of the $z$-plane, may be reduced by the transformation

**10.4.107** $W(z)=w(z)\exp\left(-\frac{1}{2}\int^z a(t,\lambda)dt\right)$

to the equation

**10.4.108**

$$w''+\varphi(z,\lambda)w=0$$

$$\varphi(z,\lambda)=b(z,\lambda)-\frac{1}{4}a^2(z,\lambda)-\frac{1}{2}\frac{d}{dz}a(z,\lambda).$$