Asymptotic Expansions for Large Orders

12.1.34

$$\mathbf{H}_\nu(z)-Y_\nu(z)\sim\frac{2(\frac{1}{2}z)^\nu}{\sqrt{\pi}\,\Gamma(\nu+\frac{1}{2})}\sum_{k=0}^{\infty}\frac{k!\,b_k}{z^{k+1}}\qquad(|\arg z|<\tfrac{1}{2}\pi,|\nu|<|z|)$$

$b_0=1,\ b_1=2\nu/z,\ b_2=6(\nu/z)^2-\frac{1}{2},\ b_3=20(\nu/z)^3-4(\nu/z)$

12.1.35

$$\mathbf{H}_\nu(z)+iJ_\nu(z)\sim\frac{2(\frac{1}{2}z)^\nu}{\sqrt{\pi}\,\Gamma(\nu+\frac{1}{2})}\sum_{k=0}^{\infty}\frac{k!\,b_k}{z^{k+1}}\qquad(|\nu|>|z|)$$

12.2. Modified Struve Function $\mathbf{L}_\nu(z)$

Power Series Expansion

12.2.1

$$\mathbf{L}_\nu(z)=-ie^{-\frac{i\nu\pi}{2}}\mathbf{H}_\nu(iz)$$

$$=(\tfrac{1}{2}z)^{\nu+1}\sum_{k=0}^{\infty}\frac{(z/2)^{2k}}{\Gamma(k+\frac{3}{2})\Gamma(k+\nu+\frac{3}{2})}$$

Integral Representations

12.2.2

$$\mathbf{L}_\nu(z)=\frac{2(z/2)^\nu}{\sqrt{\pi}\,\Gamma(\nu+\frac{1}{2})}\int_0^{\frac{\pi}{2}}\sinh(z\cos\theta)\sin^{2\nu}\theta\,d\theta\qquad(\mathscr{R}\nu>-\tfrac{1}{2})$$

12.2.3

$$I_{-\nu}(x)-\mathbf{L}_\nu(x)=\frac{2(x/2)^\nu}{\sqrt{\pi}\,\Gamma(\nu+\frac{1}{2})}\int_0^{\infty}\sin(tx)(1+t^2)^{\nu-\frac{1}{2}}\,dt\qquad(\mathscr{R}\nu<\tfrac{1}{2},x>0)$$

Recurrence Relations

12.2.4

$$\mathbf{L}_{\nu-1}-\mathbf{L}_{\nu+1}=\frac{2\nu}{z}\mathbf{L}_\nu+\frac{(z/2)^\nu}{\sqrt{\pi}\,\Gamma(\nu+\frac{3}{2})}$$

12.2.5

$$\mathbf{L}_{\nu-1}+\mathbf{L}_{\nu+1}=2\mathbf{L}'_\nu-\frac{(z/2)^\nu}{\sqrt{\pi}\,\Gamma(\nu+\frac{3}{2})}$$

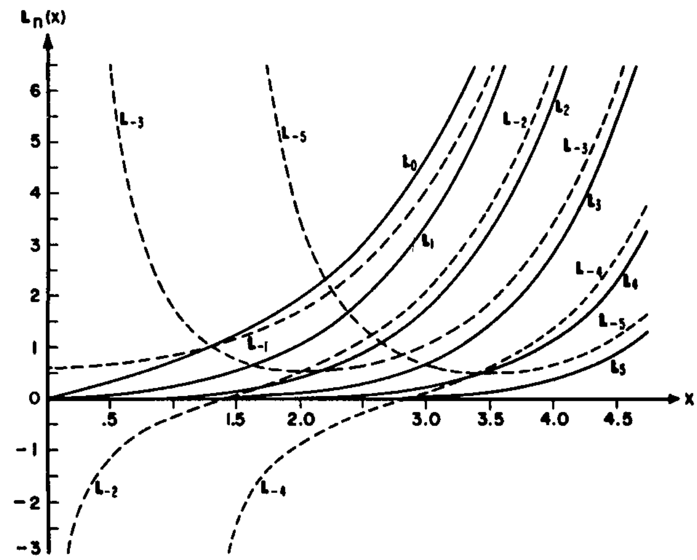

FIGURE 12.4. *Modified Struve functions.*

$\mathbf{L}_n(x),\ \pm n=0(1)5$

Asymptotic Expansion for Large $|z|$

12.2.6

$$\mathbf{L}_\nu(z)-I_{-\nu}(z)\sim\frac{1}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^{k+1}\Gamma(k+\frac{1}{2})}{\Gamma(\nu+\frac{1}{2}-k)\left(\frac{z}{2}\right)^{2k-\nu+1}}\qquad(|\arg z|<\tfrac{1}{2}\pi)$$

Integrals

12.2.7

$$\int_0^z\mathbf{L}_0(t)dt=\frac{2}{\pi}\left[\frac{z^2}{2}+\frac{z^4}{1^2\cdot3^2\cdot4}+\frac{z^6}{1^2\cdot3^2\cdot5^2\cdot6}+\cdots\right]$$

12.2.8

$$\int_0^z[I_0(t)-\mathbf{L}_0(t)]dt-\frac{2}{\pi}[\ln(2z)+\gamma]$$

$$\sim-\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{(2k)!\,(2k-1)!}{(k!)^2(2z)^{2k}}\qquad(|\arg z|<\tfrac{1}{2}\pi)$$

12.2.9

$$\int_0^z\mathbf{L}_1(t)dt=\mathbf{L}_0(z)-\frac{2}{\pi}z$$

Relation to Modified Spherical Bessel Function

12.2.10 $\mathbf{L}_{-(n+\frac{1}{2})}(z)=I_{(n+\frac{1}{2})}(z)\qquad(n\text{ an integer}\geq0)$

12.3. Anger and Weber Functions

Anger's Function

12.3.1

$$\mathbf{J}_\nu(z)=\frac{1}{\pi}\int_0^\pi\cos(\nu\theta-z\sin\theta)\,d\theta$$

12.3.2 $\mathbf{J}_n(z)=J_n(z)\qquad(n\text{ an integer})$

Weber's Function

12.3.3

$$\mathbf{E}_\nu(z)=\frac{1}{\pi}\int_0^\pi\sin(\nu\theta-z\sin\theta)\,d\theta$$

Relations Between Anger's and Weber's Function

12.3.4 $\sin(\nu\pi)\,\mathbf{J}_\nu(z)=\cos(\nu\pi)\,\mathbf{E}_\nu(z)-\mathbf{E}_{-\nu}(z)$

12.3.5 $\sin(\nu\pi)\,\mathbf{E}_\nu(z)=\mathbf{J}_{-\nu}(z)-\cos(\nu\pi)\,\mathbf{J}_\nu(z)$

Relations Between Weber's Function and Struve's Function

If n is a positive integer or zero,

12.3.6

$$\mathbf{E}_n(z)=\frac{1}{\pi}\sum_{k=0}^{\left[\frac{n-1}{2}\right]}\frac{\Gamma(k+\frac{1}{2})(\frac{1}{2}z)^{n-2k-1}}{\Gamma(n+\frac{1}{2}-k)}-\mathbf{H}_n(z)\quad*$$

12.3.7

$$\mathbf{E}_{-n}(z)=\frac{(-1)^{n+1}}{\pi}\sum_{k=0}^{\left[\frac{n-1}{2}\right]}\frac{\Gamma(n-k-\frac{1}{2})(\frac{1}{2}z)^{-n+2k+1}}{\Gamma(k+\frac{3}{2})}-\mathbf{H}_{-n}(z)\quad*$$

*See page II.