If $x=(2b-4a)[1+t/(b-2a)^{\frac23}]$, so that

$$x\sim 2b-4a$$

**13.5.19**

$$M(a,b,x)=e^{\frac12x}(b-2a)^{\frac13-b}\Gamma(b)[\mathrm{Ai}(t)\cos(a\pi)+\mathrm{Bi}(t)\sin(a\pi)+O(|\tfrac12b-a|^{-\frac23})]$$

**13.5.20**

$$U(a,b,x)=e^{\frac12x+a-\frac12b}\Gamma(\tfrac13)\pi^{-\frac12}x6^{-\frac16}\{1-t\Gamma(\tfrac56)(bx-2ax)^{-\frac13}3^{\frac13}\pi^{-\frac12}+O(|\tfrac12b-a|^{-\frac23})\}$$

If $\cos^2\theta=x/(2b-4a)$ so that $2b-4a>x>0$,

**13.5.21**

$$M(a,b,x)=\Gamma(b)\exp\{(b-2a)\cos^2\theta\}[(b-2a)\cos\theta]^{1-b}[\pi(\tfrac12b-a)\sin 2\theta]^{-\frac12}[\sin(a\pi)+\sin\{(\tfrac12b-a)(2\theta-\sin 2\theta)+\tfrac14\pi\}+O(|\tfrac12b-a|^{-1})]$$

**13.5.22**

$$U(a,b,x)=\exp[(b-2a)\cos^2\theta][(b-2a)\cos\theta]^{1-b}[(\tfrac12b-a)\sin 2\theta]^{-\frac12}\{\sin[(\tfrac12b-a)(2\theta-\sin 2\theta)+\tfrac14\pi]+O(|\tfrac12b-a|^{-1})\}$$

## 13.6. Special Cases

| | $M(a,b,z)$ | | | Relation | Function |
|---|---|---|---|---|---|
| | $a$ | $b$ | $z$ | | |
| 13.6.1 | $\nu+\frac12$ | $2\nu+1$ | $2iz$ | $\Gamma(1+\nu)e^{iz}(\frac12z)^{-\nu}J_\nu(z)$ | Bessel |
| 13.6.2 | $-\nu+\frac12$ | $-2\nu+1$ | $2iz$ | $\Gamma(1-\nu)e^{iz}(\frac12z)^{\nu}[\cos(\nu\pi)J_\nu(z)-\sin(\nu\pi)Y_\nu(z)]$ | Bessel |
| 13.6.3 | $\nu+\frac12$ | $2\nu+1$ | $2z$ | $\Gamma(1+\nu)e^{z}(\frac12z)^{-\nu}I_\nu(z)$ | Modified Bessel |
| 13.6.4 | $n+1$ | $2n+2$ | $2iz$ | $\Gamma(\frac32+n)e^{iz}(\frac12z)^{-n-\frac12}J_{n+\frac12}(z)$ | Spherical Bessel |
| 13.6.5 | $-n$ | $-2n$ | $2iz$ | $\Gamma(\frac12-n)e^{iz}(\frac12z)^{n+\frac12}J_{-n-\frac12}(z)$ | Spherical Bessel |
| 13.6.6 | $n+1$ | $2n+2$ | $2z$ | $\Gamma(\frac32+n)e^{z}(\frac12z)^{-n-\frac12}I_{n+\frac12}(z)$ * | Spherical Bessel |
| 13.6.7 | $n+\frac12$ | $2n+1$ | $-2\sqrt{i}x$ | $\Gamma(1+n)e^{-\sqrt{i}x}(\frac12\sqrt{i}x)^{-n}(\mathrm{ber}_n\,x+i\,\mathrm{bei}_n\,x)$ | Kelvin |
| 13.6.8 | $L+1-i\eta$ | $2L+2$ | $2ix$ | $e^{ix}F_L(\eta,x)x^{-L-1}/C_L(\eta)$ | Coulomb Wave |
| 13.6.9 | $-n$ | $\alpha+1$ | $x$ | $\frac{n!}{(\alpha+1)_n}L_n^{(\alpha)}(x)$ | Laguerre |
| 13.6.10 | $a$ | $a+1$ | $-x$ | $ax^{-a}\gamma(a,x)$ | Incomplete Gamma |
| 13.6.11 | $-n$ | $1+\nu-n$ | $x$ | $\frac{(n!)^{\frac12}x^{\frac12n}}{(1+\nu-n)_n}\rho_n(\nu,x)$ | Poisson-Charlier |
| 13.6.12 | $a$ | $a$ | $z$ | $e^z$ | Exponential |
| 13.6.13 | 1 | 2 | $-2iz$ | $\frac{e^{-iz}}{z}\sin z$ | Trigonometric |
| 13.6.14 | 1 | 2 | $2z$ | $\frac{e^z}{z}\sinh z$ | Hyperbolic |
| 13.6.15 | $-\frac12\nu$ | $\frac12$ | $\frac12z^2$ | $2^{-\frac12}\exp(\frac12z^2)E_\nu^{(0)}(z)$ | Weber or Parabolic Cylinder |
| 13.6.16 | $\frac12-\frac12\nu$ | $\frac32$ | $\frac12z^2$ | $\frac{\exp(\frac12z^2)}{2z}E_\nu^{(1)}(z)$ | Weber or Parabolic Cylinder |
| 13.6.17 | $-n$ | $\frac12$ | $\frac12x^2$ | $\frac{n!}{(2n)!}(-\frac12)^{-n}He_{2n}(x)$ | Hermite |
| 13.6.18 | $-n$ | $\frac32$ | $\frac12x^2$ | $\frac{n!}{(2n+1)!}(-\frac12)^{-n}\frac{1}{x}He_{2n+1}(x)$ * | Hermite |
| 13.6.19 | $\frac12$ | $\frac32$ | $-x^2$ | $\frac{\pi^{\frac12}}{2x}\operatorname{erf}x$ | Error Integral |
| 13.6.20 | $\frac12m+\frac12$ | $1+n$ | $r^2$ | $\frac{n!\,r^{-2n+m-1}}{\Gamma(\frac12m+\frac12)}e^{r^2}T(m,n,r)$ * | Toronto |

*See page II.