has three (regular) singular points $z=0, 1, \infty$. The pairs of exponents at these points are

15.5.2 $\rho_{1,2}^{(0)}=0, 1-c, \qquad \rho_{1,2}^{(1)}=0, c-a-b, \qquad \rho_{1,2}^{(\infty)}=a, b$

respectively. The general theory of differential equations of the Fuchsian type distinguishes between the following cases.

A. None of the numbers c, c—a—b; a—b is equal to an integer. Then two linearly independent solutions of **15.5.1** in the neighborhood of the singular points 0, 1, ∞ are respectively

15.5.3 $w_{1(0)}=F(a, b; c; z)=(1-z)^{c-a-b}F(c-a, c-b; c; z)$

15.5.4 $w_{2(0)}=z^{1-c}F(a-c+1, b-c+1; 2-c; z)=z^{1-c}(1-z)^{c-a-b}F(1-a, 1-b; 2-c; z)$

15.5.5 $w_{1(1)}=F(a, b; a+b+1-c; 1-z)=z^{1-c}F(1+b-c, 1+a-c; a+b+1-c; 1-z)$

15.5.6 $w_{2(1)}=(1-z)^{c-a-b}F(c-b, c-a; c-a-b+1; 1-z)=z^{1-c}(1-z)^{c-a-b}F(1-a, 1-b; c-a-b+1; 1-z)$

15.5.7 $w_{1(\infty)}=z^{-a}F(a, a-c+1; a-b+1; z^{-1})=z^{b-c}(z-1)^{c-a-b}F(1-b, c-b; a-b+1; z^{-1})$

15.5.8 $w_{2(\infty)}=z^{-b}F(b, b-c+1; b-a+1; z^{-1})=z^{a-c}(z-1)^{c-a-b}F(1-a, c-a; b-a+1; z^{-1})$

The second set of the above expressions is obtained by applying **15.3.3** to the first set.

Another set of representations is obtained by applying **15.3.4** to **15.5.3** through **15.5.8.** This gives **15.5.9–15.5.14.**

15.5.9 $w_{1(0)}=(1-z)^{-a}F\left(a, c-b; c; \frac{z}{z-1}\right)=(1-z)^{-b}F\left(b, c-a; c; \frac{z}{z-1}\right)$

15.5.10 $w_{2(0)}=z^{1-c}(1-z)^{c-a-1}F\left(a-c+1, 1-b; 2-c; \frac{z}{z-1}\right)=z^{1-c}(1-z)^{c-b-1}F\left(b-c+1, 1-a; 2-c; \frac{z}{z-1}\right)$

15.5.11 $w_{1(1)}=z^{-a}F(a, a-c+1; a+b-c+1; 1-z^{-1})=z^{-b}F(b, b-c+1; a+b-c+1; 1-z^{-1})$

15.5.12

$w_{2(1)}=z^{a-c}(1-z)^{c-a-b}F(c-a, 1-a; c-a-b+1; 1-z^{-1})=z^{b-c}(1-z)^{c-a-b}F(c-b, 1-b; c-a-b+1; 1-z^{-1})$

15.5.13 $w_{1(\infty)}=(z-1)^{-a}F\left(a, c-b; a-b+1; \frac{1}{1-z}\right)=(z-1)^{-b}F\left(b, c-a; b-a+1; \frac{1}{1-z}\right)$

15.5.14

$w_{2(\infty)}=z^{1-c}(z-1)^{c-a-1}F\left(a-c+1, 1-b; a-b+1; \frac{1}{1-z}\right)=z^{1-c}(z-1)^{c-b-1}F\left(b-c+1, 1-a; b-a+1; \frac{1}{1-z}\right)$

15.5.3 to **15.5.14** constitute Kummer's 24 solutions of the hypergeometric equation. The analytic continuation of $w_{1,2(0)}(z)$ can then be obtained by means of **15.3.3** to **15.3.9.**

B. One of the numbers a, b, c—a, c—b is an integer. Then one of the hypergeometric series for instance $w_{1,2(0)}$, **15.5.3, 15.5.4** terminates and the corresponding solution is of the form

15.5.15 $w=z^{\alpha}(1-z)^{\beta}p_n(z)$

where $p_n(z)$ is a polynomial in z of degree n. This case is referred to as the degenerate case of the hypergeometric differential equation and its solutions are listed and discussed in great detail in [15.2].

C. The number c—a—b is an integer, c nonintegral. Then **15.3.10** to **15.3.12** give the analytic continuation of $w_{1,2(0)}$ into the neighborhood of $z=1$. Similarly **15.3.13** and **15.3.14** give the analytic continuation of $w_{1,2(0)}$ into the neighborhood of $z=\infty$ in case $a-b$ is an integer but not c, subject of course to the further restrictions $c-a=0, \pm1, \pm2 \ldots$ (For a detailed discussion of all possible cases, see [15.2]).

D. The number c=1. Then **15.5.3, 15.5.4** are replaced by

15.5.16 $w_{1(0)}=F(a, b; 1; z)$