

# 17. Elliptic Integrals

L. M. MILNE-THOMSON<sup>1</sup>

## Contents

	Page
<b>Mathematical Properties</b> . . . . .	589
17.1. Definition of Elliptic Integrals . . . . .	589
17.2. Canonical Forms . . . . .	589
17.3. Complete Elliptic Integrals of the First and Second Kinds . .	590
17.4. Incomplete Elliptic Integrals of the First and Second Kinds .	592
17.5. Landen's Transformation . . . . .	597
17.6. The Process of the Arithmetic-Geometric Mean . . . . .	598
17.7. Elliptic Integrals of the Third Kind . . . . .	599
<b>Numerical Methods</b> . . . . .	600
17.8. Use and Extension of the Tables . . . . .	600
<b>References</b> . . . . .	606
<b>Table 17.1.</b> Complete Elliptic Integrals of the First and Second Kinds and the Nome $q$ With Argument the Parameter $m$ . . . . .	608
$K(m), K'(m), 15D; q(m), q_1(m), 15D; E(m), E'(m), 9D$ $m=0(.01)1$	
<b>Table 17.2.</b> Complete Elliptic Integrals of the First and Second Kinds and the Nome $q$ With Argument the Modular Angle $\alpha$ . . . . .	610
$K(\alpha), K'(\alpha), q(\alpha), q_1(\alpha), E(\alpha), E'(\alpha), 15D$ $\alpha=0^\circ(1^\circ)90^\circ$	
<b>Table 17.3.</b> Parameter $m$ With Argument $K'(m)/K(m)$ . . . . .	612
$K'(m)/K(m)=.3(.02)3, 10D$	
<b>Table 17.4.</b> Auxiliary Functions for Computation of the Nome $q$ and the Parameter $m$ . . . . .	612
$Q(m)=q_1(m)/m_1, 15D$ $L(m)=-K(m)+\frac{K'(m)}{\pi} \ln\left(\frac{16}{m_1}\right), 10D$ $m_1=0(.01).15$	
<b>Table 17.5.</b> Elliptic Integral of the First Kind $F(\varphi\backslash\alpha)$ . . . . .	613
$\alpha=0^\circ(2^\circ)90^\circ, 5^\circ(10^\circ)85^\circ, \varphi=0^\circ(5^\circ)90^\circ, 8D$	
<b>Table 17.6.</b> Elliptic Integral of the Second Kind $E(\varphi\backslash\alpha)$ . . . . .	616
$\alpha=0^\circ(2^\circ)90^\circ, 5^\circ(10^\circ)85^\circ, \varphi=0^\circ(5^\circ)90^\circ, 8D$	

<sup>1</sup> University of Arizona. (Prepared under contract with the National Bureau of Standards.)