

Reversed Series⁵ for Large $|\mathcal{P}|$

18.5.25

$$\begin{aligned}
z = & \frac{1}{2} \left[2u + c_2 u^5 + c_3 u^7 + \frac{\alpha_2^2}{3} u^9 + \frac{6\alpha_2 \alpha_3}{11} u^{11} \right. \\
& + \frac{1}{13} (3\alpha_3^2 + 5\alpha_2^3) u^{13} + \frac{\alpha_2^2 \alpha_3}{68} u^{15} + \frac{5\alpha_2}{68} (12\alpha_3^2 + 7\alpha_2^3) u^{17} \\
& + \frac{5\alpha_3}{19} (\alpha_3^2 + 7\alpha_2^3) u^{19} + \frac{\alpha_2^2}{4} (3\alpha_3^2 + 10\alpha_2^3) u^{21} \\
& + \frac{35\alpha_2 \alpha_3}{92} (9\alpha_2^2 + 4\alpha_3^2) u^{23} \\
& + \frac{7}{200} (33\alpha_2^2 + 180\alpha_2^3 \alpha_3^2 + 10\alpha_3^4) u^{25} \\
& + \frac{7\alpha_2^2 \alpha_3}{12} (11\alpha_2^2 + 10\alpha_3^2) u^{27} \\
& + \frac{3\alpha_2}{2^3 \cdot 29} (143\alpha_2^2 + 1155\alpha_2^3 \alpha_3^2 + 210\alpha_3^4) u^{29} \\
& + \frac{21\alpha_3}{2^3 \cdot 31} (143\alpha_2^2 + 220\alpha_2^3 \alpha_3^2 + 6\alpha_3^4) u^{31} \\
& + \frac{3\alpha_2^2}{2^6} (65\alpha_2^2 + 728\alpha_2^3 \alpha_3^2 + 280\alpha_3^4) u^{33} \\
& + \frac{33\alpha_2 \alpha_3}{2^3 \cdot 5 \cdot 7} (195\alpha_2^2 + 455\alpha_2^3 \alpha_3^2 + 42\alpha_3^4) u^{35} \\
& + \frac{11}{2^6 \cdot 37} (1105\alpha_2^2 + 16380\alpha_2^3 \alpha_3^2 + 10920\alpha_2^3 \alpha_3^4) \\
& + 168\alpha_3^8) u^{37} + \frac{33\alpha_2^2 \alpha_3}{2^6} (85\alpha_2^2 + 280\alpha_2^3 \alpha_3^2 + 56\alpha_3^4) u^{39} \\
& + \frac{143\alpha_2}{2^7 \cdot 41} (323\alpha_2^2 + 6120\alpha_2^3 \alpha_3^2 + 6300\alpha_2^3 \alpha_3^4 + 336\alpha_3^8) u^{41} \\
& + \frac{143\alpha_3}{2^6 \cdot 43} (1615\alpha_2^2 + 7140\alpha_2^3 \alpha_3^2 + 2520\alpha_2^3 \alpha_3^4 + 24\alpha_3^8) u^{43} \\
& \left. + O(u^{45}) \right],
\end{aligned}$$

18.5.26 where $\alpha_2 = g_2/8$ 18.5.27 $\alpha_3 = g_2/8$ 18.5.28 $u = (\mathcal{P}^{-1})^{\frac{1}{2}}$ Reversed Series for Large $|\mathcal{P}'|$ 18.5.29 $z = A_1 u + A_5 u^5 + A_7 u^7 + A_9 u^9 + \dots$ 18.5.30 where $u = (\mathcal{P}'^{1/3})^{-1} e^{i\pi/3}$ 18.5.31 $A_1 = 2^{1/3}$ 18.5.32 $A_5 = -\frac{a_2}{5} A_1^2$ 18.5.33 $A_7 = \frac{-4a_3 A_1}{7}$ 18.5.34 $A_9 = 0$ 18.5.35 $A_{11} = 8a_2 a_3 A_1^2 / 11$ 18.5.36 $A_{13} = \frac{10A_1}{39} (a_2^2 + 6a_3^2)$ 18.5.37 $A_{15} = -96a_2^2 a_3 / 175$ 18.5.38 $A_{17} = -\frac{14a_2 A_1^2}{51} (a_2^2 + 12a_3^2)$ 18.5.39 where $a_2 = g_2/6$, $a_3 = g_3/6$ Reversed Series for Large $|\zeta|$ 18.5.40 $z = u + A_5 u^5 + A_7 u^7 + A_9 u^9 + \dots$ 18.5.41 where $u = \zeta^{-1}$ 18.5.42 $A_5 = -\delta_2/5$ 18.5.43 $A_7 = -\delta_3/7$ 18.5.44 $A_9 = \delta_2^2/7$ 18.5.45 $A_{11} = 3\delta_2 \delta_3 / 11$ 18.5.46 $A_{13} = \frac{17}{1001} (-8\delta_2^3 + 7\delta_3^2)$ 18.5.47 $A_{15} = -41\delta_2^2 \delta_3 / 91$ 18.5.48 $A_{17} = \frac{\delta_2}{9163} (1349\delta_2^2 - 4116\delta_3^2)$ 18.5.49 $A_{19} = \frac{2\delta_3}{323323} (115431\delta_2^2 - 22568\delta_3^2)$ 18.5.50 where $\delta_2 = g_2/12$ 18.5.51 $\delta_3 = g_3/20$

⁵ In this and other series a choice of the value of the root has been made so that z will be in the Fundamental Rectangle (Figure 18.2), whenever the value of the given function is appropriate.