

Reduction for  $z_2$  in  $\Delta_2$ :  $z_1 = \epsilon \bar{z}_2$  is in  $\Delta_1$ .

18.13.1  $\mathcal{P}(z_2) = \epsilon^{-2} \bar{\mathcal{P}}(z_1)$

18.13.2  $\mathcal{P}'(z_2) = -\bar{\mathcal{P}}'(z_1)$

18.13.3  $\zeta(z_2) = \epsilon^{-1} \bar{\zeta}(z_1)$

18.13.4  $\sigma(z_2) = \epsilon \bar{\sigma}(z_1)$

Reduction for  $z_3$  in  $\Delta_3$ :  $z_1 = \epsilon^{-1}(2\omega' - z_3)$  is in  $\Delta_1$

18.13.5  $\mathcal{P}(z_3) = \epsilon^{-2} \mathcal{P}(z_1)$

18.13.6  $\mathcal{P}'(z_3) = \mathcal{P}'(z_1)$

18.13.7  $\zeta(z_3) = -\epsilon^{-1} \zeta(z_1) + 2\eta'$ ,  $\eta' = \zeta(\omega')$

18.13.8  $\sigma(z_3) = \epsilon \sigma(z_1) \exp [(z_3 - \omega')(2\eta')]$

Special Values and Formulas

18.13.9

$\Delta = -27$ ,  $H_1 = \sqrt{3}(4^{-1/3})\bar{\epsilon}$ ,

$H_2 = \sqrt{3}(4^{-1/3})$ ,  $H_3 = \sqrt{3}(4^{-1/3})\epsilon$

18.13.10  $m = \sin^2 15^\circ = \frac{2 - \sqrt{3}}{4}$ ,  $q = ie^{-\pi\sqrt{3}/2}$

18.13.11  $\vartheta_2(0) = Ae^{i\pi/8}$

18.13.12  $\vartheta_3(0) = Ae^{i\pi/24}$

18.13.13  $\vartheta_4(0) = Ae^{-i\pi/24}$

18.13.14

where  $A = (\omega_2/\pi)^{1/2} 2^{1/3} 3^{1/8} \approx 1.0086 67$

18.13.15  $\omega_2 = \frac{K(m)2^{1/3}}{3^{1/4}} = \frac{\Gamma^3(1/3)}{4\pi}$

Values at Half-periods

	$\mathcal{P}$	$\mathcal{P}'$	$\zeta$	$\sigma$
18.13.16 $\omega \equiv \omega_1$	$e_1 = 4^{-1/3}\epsilon^2$	0	$\eta = \epsilon\pi/2\omega_2\sqrt{3}$	$\epsilon^{-1}\sigma(\omega_2)$
18.13.17 $\omega_2$	$e_2 = 4^{-1/3}$	0	$\eta_2 = \eta + \eta' = \pi/2\omega_2\sqrt{3}$	$\frac{e^{\pi/4\sqrt{3}}(2^{1/3})}{3^{1/2}}$
18.13.18 $\omega' \equiv \omega_3$	$e_3 = 4^{-1/3}\epsilon^{-2}$	0	$\eta' = \epsilon^{-1}\pi/2\omega_2\sqrt{3}$	$\epsilon\sigma(\omega_2)$
18.13.19 $\omega_2'$	$e_2 = 4^{-1/3}$	0	$\eta_2' = -\pi i/2\omega_2 = \eta' - \eta$	$\frac{ie^{3\pi/4\sqrt{3}}(2^{1/3})}{3^{1/2}}$

Values <sup>7</sup> along  $(0, \omega_2)$

	$\mathcal{P}$	$\mathcal{P}'$	$\zeta$	$\sigma$
18.13.20 $2\omega_2/9$	$\frac{\sqrt[3]{\cos 80^\circ}}{\sqrt[3]{\cos 20^\circ} - \sqrt[3]{\cos 40^\circ}}$	$-\frac{\sqrt{3}[\sqrt[3]{\cos 20^\circ} + \sqrt[3]{\cos 40^\circ}]}{\sqrt[3]{\cos 20^\circ} - \sqrt[3]{\cos 40^\circ}}$		
18.13.21 $\omega_2/3$	$1/(2^{1/3} - 1)$	$-\sqrt{3}(2^{1/3} + 1)/(2^{1/3} - 1)$	$\frac{\eta_2}{3} + \frac{\sqrt{3}(2^{2/3} + 2 + 2^{4/3})}{6}$	$\frac{e^{\pi/36\sqrt{3}}}{3^{1/6}} \sqrt[4]{\frac{2^{1/3} - 1}{2^{1/3} + 1}}$
18.13.22 $4\omega_2/9$	$\frac{\sqrt[3]{\cos 40^\circ}}{\sqrt[3]{\cos 20^\circ} - \sqrt[3]{\cos 80^\circ}}$	$-\frac{\sqrt{3}[\sqrt[3]{\cos 20^\circ} + \sqrt[3]{\cos 80^\circ}]}{\sqrt[3]{\cos 20^\circ} - \sqrt[3]{\cos 80^\circ}}$		
18.13.23 $\omega_2/2$	$e_2 + H_2$	$-3^{3/4}\sqrt{2 + \sqrt{3}}$	$(\pi/4\omega_2\sqrt{3}) + (3^{1/4}\sqrt{2 + \sqrt{3}}/2^{4/3})$	$\frac{e^{\pi/16\sqrt{3}}(2^{1/12})}{3^{1/4}\sqrt{2 + \sqrt{3}}}$
18.13.24 $2\omega_2/3$	1	$-\sqrt{3}$	$\frac{2}{3}(\eta_2) + 3^{-1/2}$	$e^{\pi/9\sqrt{3}}/3^{1/6}$
18.13.25 $8\omega_2/9$	$\frac{\sqrt[3]{\cos 20^\circ}}{\sqrt[3]{\cos 40^\circ} + \sqrt[3]{\cos 80^\circ}}$	$-\frac{\sqrt{3}[\sqrt[3]{\cos 40^\circ} - \sqrt[3]{\cos 80^\circ}]}{\sqrt[3]{\cos 40^\circ} + \sqrt[3]{\cos 80^\circ}}$		

<sup>7</sup> Values at  $2\omega_2/9$ ,  $4\omega_2/9$  and  $8\omega_2/9$  from [18.14].