

Table 18.3

INVARIANTS AND VALUES AT HALF-PERIODS
(Non-Negative Discriminant—Unit Real Half-Period)

$a = \omega'/i$	g_2	g_3	$e_1 = \mathcal{P}(1)$	$e_3 = \mathcal{P}(\omega')$	$\eta = \zeta(1)$	$\eta'/i = \zeta(\omega')/i$
1.00	11.81704 500	0.00000 000	1.71879 64	-1.71879 64	0.78539 816	-0.78539 82
1.02	11.37372 384	0.55318 992	1.71005 96	-1.66138 15	0.78979 718	-0.76520 32
1.04	10.98419 107	1.03485 699	1.70235 77	-1.60783 69	0.79367 192	-0.74537 75
1.06	10.64177 347	1.45484 521	1.69556 79	-1.55787 59	0.79708 535	-0.72588 58
1.08	10.34065 794	1.82151 890	1.68958 18	-1.51123 63	0.80009 279	-0.70669 61
1.10	10.07577 364	2.14201 000	1.68430 01	-1.46767 83	0.80274 283	-0.68777 92
1.12	9.84269 185	2.42241 937	1.67965 48	-1.42698 19	0.80507 817	-0.66910 88
1.14	9.63754 049	2.66798 153	1.67554 80	-1.38894 48	0.80713 637	-0.65066 09
1.16	9.45693 072	2.88320 000	1.67193 04	-1.35338 12	0.80895 045	-0.63241 38
1.18	9.29789 413	3.07195 918	1.66874 05	-1.32011 96	0.81054 949	-0.61434 79
1.20	9.15782 851	3.23761 717	1.66592 77	-1.28900 20	0.81195 906	-0.59644 54
1.22	9.03445 117	3.38308 317	1.66344 74	-1.25988 23	0.81320 168	-0.57869 03
1.24	8.92575 843	3.51088 223	1.66126 03	-1.23262 55	0.81429 717	-0.56106 78
1.26	8.82999 055	3.62320 977	1.65933 17	-1.20710 65	0.81526 299	-0.54356 50
1.28	8.74560 138	3.72197 756	1.65763 09	-1.18320 95	0.81611 453	-0.52616 97
1.30	8.67123 169	3.80885 265	1.65613 11	-1.16082 70	0.81686 533	-0.50887 14
1.32	8.60568 628	3.88529 056	1.65480 86	-1.13985 91	0.81752 732	-0.49166 03
1.34	8.54791 374	3.95256 351	1.65364 22	-1.12021 33	0.81811 103	-0.47452 75
1.36	8.49698 890	4.01178 462	1.65261 37	-1.10180 31	0.81862 572	-0.45746 53
1.38	8.45209 746	4.06392 870	1.65170 67	-1.08454 85	0.81907 958	-0.44046 65
1.40	8.41252 263	4.10985 014	1.65090 68	-1.06837 47	0.81947 977	-0.42352 46
1.42	8.37763 305	4.15029 819	1.65020 13	-1.05321 20	0.81983 269	-0.40663 39
1.44	8.34687 283	4.18593 045	1.64957 92	-1.03899 58	0.82014 389	-0.38978 91
1.46	8.31975 228	4.21732 438	1.64903 06	-1.02566 55	0.82041 831	-0.37298 56
1.48	8.29583 997	4.24498 728	1.64854 68	-1.01316 45	0.82066 031	-0.35621 91
1.50	8.27475 580	4.26936 502	1.64812 02	-1.00144 04	0.82087 370	-0.33948 58
1.52	8.25616 484	4.29084 965	1.64774 39	-0.99044 37	0.82106 191	-0.32278 22
1.54	8.23977 191	4.30978 602	1.64741 20	-0.98012 84	0.82122 787	-0.30610 54
1.56	8.22531 684	4.32647 752	1.64711 94	-0.97045 19	0.82137 423	-0.28945 25
1.58	8.21257 036	4.34119 120	1.64686 13	-0.96137 37	0.82150 329	-0.27282 11
1.60	8.20133 033	4.35416 210	1.64663 38	-0.95285 64	0.82161 711	-0.25620 90
1.65	8.17870 308	4.38026 291	1.64617 54	-0.93379 17	0.82184 628	-0.21475 00
1.70	8.16217 907	4.39931 441	1.64584 08	-0.91752 88	0.82201 364	-0.17337 32
1.75	8.15011 147	4.41322 294	1.64559 63	-0.90365 18	0.82213 589	-0.13205 85
1.80	8.14129 812	4.42337 818	1.64541 78	-0.89180 82	0.82222 516	-0.09079 10
1.85	8.13486 127	4.43079 368	1.64528 73	-0.88169 76	0.82229 038	-0.04955 91
1.90	8.13016 001	4.43620 896	1.64519 21	-0.87306 52	0.82233 800	-0.00835 41
1.95	8.12672 634	4.44016 375	1.64512 25	-0.86569 37	0.82237 281	+0.03233 07
2.00	8.12421 844	4.44305 205	1.64507 17	-0.85939 82	0.82239 820	0.07400 01
2.05	8.12238 671	4.44516 152	1.64503 45	-0.85402 10	0.82241 676	0.11515 80
2.10	8.12104 883	4.44670 219	1.64500 74	-0.84942 78	0.82243 032	0.15630 73
2.15	8.12007 164	4.44782 746	1.64498 76	-0.84550 41	0.82244 022	0.19745 01
2.20	8.11935 791	4.44864 934	1.64497 32	-0.84215 20	0.82244 745	0.23858 81
2.25	8.11883 660	4.44924 963	1.64496 26	-0.83928 80	0.82245 274	0.27972 23
2.30	8.11845 583	4.44968 808	1.64495 49	-0.83684 11	0.82245 659	0.32085 38
2.4	8.11797 459	4.45024 222	1.64494 51	-0.83296 37	0.82246 146	0.40311 12
2.5	8.11771 785	4.45053 785	1.64494 00	-0.83013 28	0.82246 406	0.48536 38
2.6	8.11758 087	4.45069 555	1.64493 71	-0.82806 54	0.82246 546	0.56761 39
2.7	8.11750 782	4.45077 969	1.64493 57	-0.82655 58	0.82246 619	0.64986 24
2.8	8.11746 884	4.45082 457	1.64493 49	-0.82545 33	0.82246 659	0.73211 01
2.9	8.11744 804	4.45084 852	1.64493 45	-0.82464 81	0.82246 680	0.81435 74
3.0	8.11743 694	4.45086 130	1.64493 43	-0.82406 01	0.82246 691	0.89660 44
3.1	8.11743 103	4.45086 811	1.64493 42	-0.82363 06	0.82246 698	0.97885 13
3.2	8.11742 787	4.45087 174	1.64493 41	-0.82331 68	0.82246 701	1.06109 81
3.3	8.11742 619	4.45087 368	1.64493 41	-0.82308 78	0.82246 702	1.14334 48
3.4	8.11742 529	4.45087 472	1.64493 41	-0.82292 04	0.82246 703	1.22559 16
3.5	8.11742 481	4.45087 528	1.64493 41	-0.82279 82	0.82246 703	1.30783 83
3.6	8.11742 455	4.45087 556	1.64493 41	-0.82270 89	0.82246 703	1.39008 50
3.7	8.11742 441	4.45087 572	1.64493 41	-0.82264 37	0.82246 704	1.47233 17
3.8	8.11742 434	4.45087 581	1.64493 41	-0.82259 61	0.82246 704	1.55457 84
3.9	8.11742 430	4.45087 585	1.64493 41	-0.82256 13	0.82246 704	1.63682 51
4.0	8.11742 426	4.45087 587	1.64493 41	-0.82253 59	0.82246 704	1.71907 18
∞	8.11742 426	4.45087 590	1.64493 41	-0.82246 70	0.82246 704	∞
$\Delta=0$	$\begin{bmatrix} (-3)7 \\ 8 \end{bmatrix}$	$\begin{bmatrix} (-3)9 \\ 8 \end{bmatrix}$	$\begin{bmatrix} (-4)1 \\ 5 \end{bmatrix}$	$\begin{bmatrix} (-4)5 \\ 6 \end{bmatrix}$	$\begin{bmatrix} (-5)7 \\ 6 \end{bmatrix}$	$\begin{bmatrix} (-5)5 \\ 5 \end{bmatrix}$

For $a=1$: $g_2 = \omega^4$, $g_3 = 0$, $e_1 = \omega^2/2$, $e_3 = -\omega^2/2$, $\eta = \pi/4$, $\eta'/i = -\pi/4$.
 For $a = \infty$: $g_2 = \pi^4/12$, $g_3 = \pi^6/216$, $e_1 = \pi^2/6$, $e_3 = -\pi^2/12$, $\eta = \pi^2/12$, $\eta'/i = \infty$.
 ($\omega = 1.85407 4677$ is the real half-period in the Lemniscatic case 18.14.)

For $4 < a < \infty$, to obtain η' use Legendre's relation $\eta' = \eta\omega' - \pi i/2$.
 To obtain the corresponding values of tabulated quantities when the real half-period $\omega \neq 1$, multiply g_2 by ω^{-4} , g_3 by ω^{-6} , e_i by ω^{-2} and η by ω^{-1} .