

19. Parabolic Cylinder Functions

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The author acknowledges permission from H.M. Stationery Office to draw freely from [19.11] the material in the introduction, and the tabular values of $W(a, x)$ for $a = -5(1)5, \pm x = 0(.1)5$. Other tables of $W(a, x)$ and the tables of $U(a, x)$ and $V(a, x)$ were prepared on EDSAC 2 at the University Mathematical Laboratory, Cambridge, England, using a program prepared by Miss Joan Walsh for solution of general second order linear homogeneous differential equations with quadratic polynomial coefficients. The auxiliary tables were prepared at the Computation Laboratory of the National Bureau of Standards.

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