Polynomial Approximations [7]

4.3.96 $0 \le x \le \frac{\pi}{2}$

$$\frac{\sin x}{x} = 1 + a_2 x^2 + a_4 x^4 + \epsilon(x)$$

$$|\epsilon(x)| \le 2 \times 10^{-4}$$

$a_2 = -.16605$ $a_4 = .00761$

4.3.97 $0 \le x \le \frac{\pi}{2}$

$$\frac{\sin x}{x} = 1 + a_2 x^2 + a_4 x^4 + a_6 x^6 + a_8 x^8 + a_{10} x^{10} + \epsilon(x)$$

$$|\epsilon(x)| \le 2 \times 10^{-9}$$

$a_2 = -.16666\ 66664$ $a_8 = .00000\ 27526$
$a_4 = .00833\ 33315$ $a_{10} = -.00000\ 00239$
$a_6 = -.00019\ 84090$

4.3.98 $0 \le x \le \frac{\pi}{2}$

$$\cos x = 1 + a_2 x^2 + a_4 x^4 + \epsilon(x)$$

$$|\epsilon(x)| \le 9 \times 10^{-4}$$

$a_2 = -.49670$ $a_4 = .03705$

4.3.99 $0 \le x \le \frac{\pi}{2}$

$$\cos x = 1 + a_2 x^2 + a_4 x^4 + a_6 x^6 + a_8 x^8 + a_{10} x^{10} + \epsilon(x)$$

$$|\epsilon(x)| \le 2 \times 10^{-9}$$

$a_2 = -.49999\ 99963$ $a_8 = .00002\ 47609$
$a_4 = .04166\ 66418$ $a_{10} = -.00000\ 02605$
$a_6 = -.00138\ 88397$

4.3.100 $0 \le x \le \frac{\pi}{4}$

$$\frac{\tan x}{x} = 1 + a_2 x^2 + a_4 x^4 + \epsilon(x)$$

$$|\epsilon(x)| \le 1 \times 10^{-3}$$

$a_2 = .31755$ $a_4 = .20330$

4.3.101 $0 \le x \le \frac{\pi}{4}$

$$\frac{\tan x}{x} = 1 + a_2 x^2 + a_4 x^4 + a_6 x^6 + a_8 x^8 + a_{10} x^{10} + a_{12} x^{12} + \epsilon(x)$$

$$|\epsilon(x)| \le 2 \times 10^{-8}$$

$a_2 = .33333\ 14036$ $a_8 = .02456\ 50893$
$a_4 = .13339\ 23995$ $a_{10} = .00290\ 05250$
$a_6 = .05337\ 40603$ $a_{12} = .00951\ 68091$

4.3.102 $0 \le x \le \frac{\pi}{4}$

* $x \cot x = 1 + a_2 x^2 + a_4 x^4 + \epsilon(x)$

$$|\epsilon(x)| \le 3 \times 10^{-5}$$

$a_2 = -.332867$ $a_4 = -.024369$

4.3.103 $0 \le x \le \frac{\pi}{4}$

$$x \cot x = 1 + a_2 x^2 + a_4 x^4 + a_6 x^6 + a_8 x^8 + a_{10} x^{10} + \epsilon(x)$$

$$|\epsilon(x)| \le 4 \times 10^{-10}$$

$a_2 = -.33333\ 33410$ $a_8 = -.00020\ 78504$
$a_4 = -.02222\ 20287$ $a_{10} = -.00002\ 62619$
$a_6 = -.00211\ 77168$

Approximations in Terms of Chebyshev Polynomials [8]

4.3.104 $-1 \le x \le 1$

$T_n^*(x) = \cos n\theta$, $\cos\theta = 2x - 1$ (see chapter 22)

$$\sin \tfrac{1}{2}\pi x = x \sum_{n=0}^{\infty} A_n T_n^*(x^2) \qquad \cos \tfrac{1}{2}\pi x = \sum_{n=0}^{\infty} A_n T_n^*(x^2)$$

n	A_n	n	A_n
0	1.27627 8962	0	.47200 1216
1	−.28526 1569	1	−.49940 3258
2	.00911 8016	2	.02799 2080
3	−.00013 6587	3	−.00059 6695
4	.00000 1185	4	.00000 6704
5	−.00000 0007	5	−.00000 0047

[7] The approximations **4.3.96** to **4.3.103** are from B. Carlson, M. Goldstein, Rational approximation of functions, Los Alamos Scientific Laboratory LA-1943, Los Alamos, N. Mex., 1955 (with permission).

[8] The approximations **4.3.104** are from C. W. Clenshaw, Polynomial approximations to elementary functions, Math. Tables Aids Comp. **8**, 143–147 (1954) (with permission).

*See page II.