

Then from 4.1.24

$$\begin{aligned} \ln 1131.718 &= (.00063 \ 4836) - \frac{1}{2}(.00063 \ 4836)^2 \\ &+ \ln 1.131 + 3 \ln 10 = .00063 \ 4836 - .00000 \ 0202 \\ &+ .12310 \ 2197 + 6.90775 \ 5279 = 7.03149 \ 211. \end{aligned}$$

Example 6.

Compute $\ln x$ working with 16D for
 $x = 1.38967 \ 12458 \ 179231$.

Since $\frac{x}{1.389} = 1.00048 \ 32583 \ 282384 = 1 + a$, using
4.1.24 and Table 4.2 we compute successively

$$\begin{aligned} a &= .00048 \ 32583 \ 282384 \\ -\frac{a^2}{2} &= - . \quad 1167 \ 693059 \\ \frac{a^3}{3} &= . \quad 376199 \\ -\frac{a^4}{4} &= - . \quad 136 \\ \ln(1+a) &= .00048 \ 31415 \ 965388 \\ \ln 1.389 &= .32858 \ 40637 \ 722067 \\ \ln x &= .32906 \ 72053 \ 687455. \end{aligned}$$

Example 7.

Compute the principal value of $\ln(\pm 2 \pm 3i)$.
From 4.1.2, 4.1.3 and Tables 4.2 and 4.14.

$$\begin{aligned} \ln(2+3i) &= \frac{1}{2} \ln(2^2+3^2) + i \arctan \frac{3}{2} \\ &= 1.282475 + i(.982794) \\ \ln(-2+3i) &= \frac{1}{2} \ln 13 + i \left(\pi - \arctan \frac{3}{2} \right) \\ &= 1.282475 + i(2.158799) \\ \ln(-2-3i) &= \frac{1}{2} \ln 13 + i \left(-\pi + \arctan \frac{3}{2} \right) \\ &= 1.282475 - i(2.158799) \\ \ln(2-3i) &= \frac{1}{2} \ln 13 + i \left(-\arctan \frac{3}{2} \right) \\ &= 1.282475 - i(.982794). \end{aligned}$$

Example 8.

Compute $(.227)^{.69}$ to 7D.
Using 4.2.7 and Tables 4.2 and 4.4,

$$\begin{aligned} (.227)^{.69} &= e^{.69 \ln(.227)} = e^{.69(-1.48280 \ 5262)} \\ &= e^{-1.02313 \ 5631} = .35946 \ 60. \end{aligned}$$

Example 9.

Compute $e^{4.99728 \ 69}$ to 7S.
Using 4.2.18 and Table 4.4,

$$e^{4.99728 \ 69} = e^{4.9} e^{.09728 \ 69}$$

Linear interpolation gives $e^{.09728 \ 69} = 1.10217 \ 6$
with an error of 1×10^{-7} ,

$$e^{4.99728 \ 69} = (134.28978)(1.10217 \ 67) = 148.0111.$$

Example 10.

Compute e^x to 18D for

$$x = .86725 \ 13489 \ 24685 \ 12693.$$

Let $a = x - .867$. Using 4.2.1, compute successively

$$\begin{aligned} &1.00000 \ 00000 \ 00000 \ 00000 \\ a &= .00025 \ 13489 \ 24685 \ 12693 \\ \frac{a^2}{2!} &= . \quad 315 \ 88140 \ 97019 \\ \frac{a^3}{3!} &= . \quad 2646 \ 54842 \\ \frac{a^4}{4!} &= . \quad 16630 \end{aligned}$$

$$e^a = 1.00025 \ 13805 \ 15472 \ 81184$$

$$e^{.867} = 2.37976 \ 08513 \ 29496 \ 863 \text{ from Table 4.}$$

$$e^a e^{.867} = e^x = 2.38035 \ 90768 \ 39006 \ 089.$$

Example 11.

Compute e^{648} to 7S.

Let $n = \frac{x}{\ln 10}$ and $d =$ the decimal part of $\frac{x}{\ln 10}$.

Then

$$\begin{aligned} \exp x &= \exp \left(\frac{x}{\ln 10} \ln 10 \right) = \exp [(n+d) \ln 10] \\ &= \exp (\ln 10^n) \exp (d \ln 10) \\ &= 10^n \exp (d \ln 10) \end{aligned}$$

From Table 4.4

$$e^{648} = \exp \left(\frac{648}{\ln 10} \ln 10 \right) = \exp (281.42282 \ 42 \ln 10)$$

$$= 10^{281} \exp (.42282 \ 42 \ln 10) = 10^{281} \exp .97358 \ 8'$$

$$= 10^{281} (2.647428) = (281)2.647428.$$

Example 12.

Compute e^{-x} for $x = .75$ using the expansion in Chebyshev polynomials.