Following the procedure in [4.3] we have from **4.2.48**

$$e^{-x}=\sum_{k=0}^{7}A_kT_k^*(x)$$

* where $T_k^*(x)$ are the Chebyshev polynomials defined in chapter **22**. Assuming $b_8=b_9=0$ we generate b_k, $k=7, 6, 5, \ldots 0$ from the recurrence relation

$$b_k=(4x-2)b_{k+1}-b_{k+2}+A_k$$

k	b_k
7	−.00000 0015
6	.00000 0400
5	−.00000 9560
4	.00018 9959
3	−.00300 9164
2	.03550 4993
1	−.27432 7449
0	.33520 2828

since $f(x)=b_0-(2x-1)b_1$,

$$e^{-.75}=.33520\ 2828-(.5)(-.27432\ 7449)$$
$$=.47236\ 6553.$$

Example 13.

Express 38°42′32″ in radians to 6D.

$$1^\circ=.01745\ 32925\ 19943\ 29577\ \text{r}$$
$$1'=.00029\ 08882\ 08665\ 72159\ 62\ \text{r}$$
$$1''=.00000\ 48481\ 36811\ 09535\ 9936\ \text{r}$$

Therefore

$$38^\circ=.66322\ 51\ \text{r}$$
$$42'=.01221\ 73\ \text{r}$$
$$32''=.00015\ 51\ \text{r}$$
$$38^\circ42'32''=.675598\ \text{r}.$$

Example 14.

Express $x=1.6789$ radians in degrees, minutes and seconds to the nearest tenth of a second.

From **Table 1.1** giving the mathematical constants we have

$$1\ \text{r}=\frac{180^\circ}{\pi}=57.29577\ 95130^\circ\ldots$$
$$1.6789\ \text{r}=96.19388^\circ$$
$$.19388^\circ\times 60=11.633'$$
$$.633'\times 60=38.0''$$
$$1.6789\ \text{r}=96^\circ11'38.0''.$$

Example 15.

Compute sin x and cos x for $x=2.317$ to 7D.
From **4.3.44** and **Table 4.6**

$$\sin(2.317)=\sin(\pi-2.317)=\sin(.82459\ 2654)$$
$$=.73427\ 12$$
$$\cos(2.317)=\cos(\pi-2.317)=-\cos(.82459\ 2654)$$
$$=-.67885\ 60.$$

Linear interpolation for $x=.82459\ 2654$ gives an error of 9×10^{-8}.

Example 16.

Compute sin x for $x=12.867$ to 8D.
From **4.3.16** and **Tables 4.6** and **4.8**

$$\sin(12.867)=\sin 12\cos .867+\cos 12\sin .867$$
$$=.29612\ 142.$$

The method of reduction to an angle in the first quadrant which was given in **Example 15** may also be used.

Example 17.

Compute sin x to 19D for

$$x=.86725\ 13489\ 24685\ 12693.$$

Let $\alpha=.867$, $\beta=x-\alpha$. From **4.3.16** and **Table 4.6**

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\sin\alpha=.76239\ 10208\ 07866\ 22598$$
$$\cos\alpha=.64711\ 66288\ 94312\ 75010$$

With the series expansions for sin β and cos β we compute successively

	1.00000	00000	00000	00000
$-\frac{\beta^2}{2!}=-$ ·		315	88140	97019
$\frac{\beta^4}{4!}=$ ·				16630
$\cos\beta=$	.99999	99684	11859	19611
$\beta=$	.00025	13489	24685	12693
$-\frac{\beta^3}{3!}=-$ ·			2646	54842
$\frac{\beta^5}{5!}=$ ·				1
$\sin\beta=$	.00025	13489	22038	57852
$\sin\alpha\cos\beta=$	.76239	09967	25351	31308
$\cos\alpha\sin\beta=$	.00016	26520	67105	82436
$\sin x=$	.76255	36487	92457	1374

*See page 11.