This procedure is equivalent to interpolation with Taylor's formula **3.6.4.**

Example 18.

In the plane triangle ABC, $a=123$, $B=29°16'$, $c=321$; find A, b.

$$b^2=a^2+c^2-2ac\cos B=(123)^2+(321)^2 -2(123)(321)\cos 29°16'$$

$$b=221.99934\ 00$$

$$\sin A=\frac{a\sin B}{b}=\frac{(123)(.48887\ 50196)}{221.99934\ 00}=.27086\ 39918$$

$$A=15°42'56.469''.$$

Example 19.

In the plane triangle ABC, $a=4$, $b=7$, $c=9$, find A, B, and C.

$$\cos A=\frac{c^2+b^2-a^2}{2bc}=\frac{81+49-16}{2\cdot 7\cdot 9}=\frac{114}{126}=.90476\ 1905$$

$$A=.43997\ 5954=25°12'31.6''$$

$$\sin A=.42591\ 7709$$

$$\sin B=\frac{7(.42591\ 7709)}{4},\quad B=.84106\ 8670 =48°11'22.9''$$

$$\sin C=\frac{9(.42591\ 7709)}{4},\quad C=1.86054\ 803 =106°36'5.6''$$

where the supplementary angle must be chosen for C. As a check we get $A+B+C=180°00'.1''$.

Example 20.

Compute $\cot x$ for $x=.4589$ to 6D.

Since $x<.5$, using **Table 4.9** with interpolation in $(x^{-1}-\cot x)$, we find $\frac{1}{.4589}-\cot(.4589)=.155159$. Therefore $\cot(.4589)=2.179124-.155159=2.023965$.

Example 21.

Compute $\arcsin x$ for $x=.99511$.

For $x>.95$, using **Table 4.14** with interpolation in the auxiliary function $f(x)$ we find

$$\arcsin x=\frac{\pi}{2}-[2(1-x)]^{\frac{1}{2}}f(x)$$

$$\arcsin(.99511)=\frac{\pi}{2}-[2(.00489)]^{\frac{1}{2}}f(.99511)$$

$$=1.57079\ 6327-(.09889\ 388252)(1.00040\ 7951)$$

$$=1.47186\ 2100.$$

Example 22.

Compute arctan 20 and arccot 20 to 9D. Using **4.4.5, 4.4.8,** and **Table 4.14** *

$$\arctan 20=\frac{\pi}{2}-\arctan 1/20=1.52083\ 7931$$

$$\text{arccot}\ 20=\frac{\pi}{2}-\arctan 20=\arctan .05=.04995\ 8396.$$

Example 23.

Express $z=3+9i$ in polar form.

$z=x+iy=re^{i\theta}$, where $r=(x^2+y^2)^{\frac{1}{2}}$,

$\theta=\arctan\frac{y}{x}+2\pi k$, k is an integer. For $k=0$, $r=(3^2+9^2)^{\frac{1}{2}}=\sqrt{90}=9.486833$

$$\theta=\arctan 9/3=\arctan 3=1.24904\ 58.$$

Thus $3+9i=9.486833\exp(1.24904\ 58i)$.

Example 24.

Compute $\arctan x$ for $x=1/3$ to 12D.

From **4.4.34** and **4.4.42** we have

$$\arctan x=\arctan(x_0+h)$$

$$=\arctan x_0+\arctan\frac{h}{1+x_0h+x_0^2}$$

$$=\arctan x_0+\left(\frac{h}{1+x_0h+x_0^2}\right)-\frac{1}{3}\left(\frac{h}{1+x_0h+x_0^2}\right)^3+\ldots.$$

We have

$x=\frac{1}{3}=.33333\ 33333\ 33$ so that $h=.00033\ 33333\ 33$ and, from **Table 4.14,** $\arctan x_0=\arctan .333=.32145\ 05244\ 03$. Since $\frac{h}{1+x_0h+x_0^2}=.00030\ 00300\ 03$ we get

$$\arctan x=.32145\ 05244\ 03+.00030\ 00300\ 03 -.00000\ 00000\ 09$$

$$=.32175\ 05543\ 97.$$

If x is given in the form b/a it is convenient to use **4.4.34** in the form

$$\arctan\frac{b}{a}=\arctan x_0+\arctan\frac{b-ax_0}{a+bx_0}.$$

In the present example we get

$$\arctan\frac{1}{3}=\arctan .333+\arctan\frac{1}{3333}.$$

*See page II.