26.6.9 then

$$F_p(\nu_1, \nu_2)=\frac{1}{F_{1-p}(\nu_2, \nu_1)}$$

Relation to Student's *t*-Distribution Function (See 26.7)

26.6.10 $Q(F|\nu_1=1, \nu_2)=1-A(t|\nu_2) \qquad t=\sqrt{F}$

Limiting Forms

26.6.11

$$\lim_{\nu_2\to\infty} Q(F|\nu_1, \nu_2)=Q(\chi^2|\nu_1), \qquad \chi^2=\nu_1 F$$

26.6.12

$$\lim_{\nu_1\to\infty} Q(F|\nu_1, \nu_2)=P(\chi^2|\nu_2), \qquad \chi^2=\frac{\nu_2}{F}$$

Approximations

26.6.13

$$Q(F|\nu_1, \nu_2)\approx Q(x), \qquad x=\frac{F-\frac{\nu_2}{\nu_2-2}}{\frac{\nu_2}{\nu_2-2}\sqrt{\frac{2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-4)}}}$$

(ν_1 and ν_2 large)

26.6.14

$$Q(F|\nu_1, \nu_2)\approx Q(x), \qquad x=\frac{\sqrt{(2\nu_2-1)\frac{\nu_1}{\nu_2}F}-\sqrt{2\nu_1-1}}{\sqrt{1+\frac{\nu_1}{\nu_2}F}}$$

26.6.15

$$Q(F|\nu_1, \nu_2)\approx Q(x), \qquad x=\frac{F^{1/3}\left(1-\frac{2}{9\nu_2}\right)-\left(1-\frac{2}{9\nu_1}\right)}{\sqrt{\frac{2}{9\nu_1}+F^{2/3}\frac{2}{9\nu_2}}}$$

Approximation to the Inverse Function

26.6.16 If $Q(F_p|\nu_1, \nu_2)=p$, then

$F_p\approx e^{2w}$ where w is given by **26.5.22**, with

$$\nu_1=2b, \; \nu_2=2a$$

Non-Central *F*-Distribution Function

26.6.17

$$P(F'|\nu_1,\nu_2,\lambda)=\int_0^{F'} p(t|\nu_1,\nu_2,\lambda)dt=1-Q(F'|\nu_1,\nu_2,\lambda)$$

where

$$p(t|\nu_1,\nu_2,\lambda)=\sum_{j=0}^{\infty} e^{-\lambda/2}\frac{(\lambda/2)^j}{j!}\frac{(\nu_1+2j)^{\frac{\nu_1+2j}{2}}\nu_2^{\nu_2/2}}{B\left(\frac{\nu_1+2j}{2},\frac{\nu_2}{2}\right)}\times t^{\frac{\nu_1+2j-2}{2}}[\nu_2+(\nu_1+2j)t]^{-(\nu_1+2j+\nu_2)/2}$$

and $\lambda\geq 0$ is termed the non-centrality parameter.

Relation of Non-Central *F*-Distribution Function to Other Functions

	Function	*Relation*
26.6.18	*F*-distribution	$P(F'\|\nu_1,\nu_2,\lambda)=\sum_{j=0}^{\infty} e^{-\lambda/2}\frac{(\lambda/2)^j}{j!}P(F'\|\nu_1+2j,\nu_2)$ $P(F'\|\nu_1,\nu_2,\lambda=0)=P(F'\|\nu_1,\nu_2)$
26.6.19	Non-central *t*-distribution	$P(F'\|\nu_1=1,\nu_2,\lambda)=P(t'\|\nu,\delta), t'=\sqrt{F'}, \nu=\nu_2, \delta=\sqrt{\lambda}$
26.6.20	Incomplete Beta function	$P(F'\|\nu_1,\nu_2)=\sum_{j=0}^{\infty} e^{-\lambda/2}\frac{(\lambda/2)^j}{j!}I_x\left(\frac{\nu_1}{2}+j,\frac{\nu_2}{2}\right),$ $x=\frac{\nu_1 F'}{\nu_1 F'+\nu_2}$ *
26.6.21	Confluent hypergeometric function	$P(F'\|\nu_1,\nu_2,\lambda)=\sum_{i=0}^{\frac{\nu_2}{2}-1}\frac{2e^{-\lambda/2}}{(\nu_1+\nu_2)B\left(\frac{\nu_1}{2}+i+1,\frac{\nu_2}{2}-i\right)}\times$ $x^{\frac{\nu_1}{2}+1}(1-x)^{\frac{\nu_2}{2}-i-1}M\left(\frac{\nu_1+\nu_2}{2},\frac{\nu_1}{2}+i+1,\frac{\lambda x}{2}\right)$ $\left(\nu_2 \text{ even and } x=\frac{\nu_2}{\nu_1 F'+\nu_2}\right)$