deviates from the distribution function $F(y)$, can be generated (approximately) by setting $y=z+w=z+\Delta\left(u-\frac{1}{2}\right)$. This is simply an approximate decomposition of the continuous random variable into the sum of a discrete and continuous random variable. The discrete variable can be generated quickly by the method described previously. The smaller the value of Δ the better will be the approximation. Each number can be generated by using the leading digits of U to generate the discrete random variable Z and the remaining digits forming a uniformly distributed deviate having (0,1) domain.

4. Acceptance-Rejection Methods

In what follows the random variable Y will be assumed to have finite domain (a, b). If the domain is infinite, it must be truncated for computational purposes at (say) the points a and b. Then the resulting random deviates will only have this truncated domain.

a) Let f be the maximum of $f(y)$. Then the procedure for generating random deviates is: (1) generate a pair of uniform deviates U_1, U_2; (2) compute a point $y=a+(b-a)u_2$ in (a, b); (3) if $u_1<f(y)/f$ accept y as the random deviate, otherwise reject the pair (u_1, u_2) and start again. The acceptance ratio of deviates actually produced is $[(b-a)f]^{-1}$. Hence the acceptance ratio decreases as the domain increases. One way to increase the acceptance ratio is to divide the interval (a, b) into mutually exclusive sub-intervals and then carry out the acceptance-rejection process. For this purpose let the interval (a, b) be divided into k sub-intervals such that the end points of the jth interval are (ξ_{j-1}, ξ_j) with $\xi_0=a$, $\xi_k=b$ and $\int_{\xi_{j-1}}^{\xi_j} f(y)dy=p_j$; further let the maximum of $f(y)$ in the jth interval be f_j. Then to generate random deviates from $f(y)$, generate n pairs of deviates $(u_{1s}, u_{2s})s=1, 2, \ldots, n$. Assign $[np_j]$ such pairs to the jth interval and compute $y_j=\xi_{j-1}+(\xi_j-\xi_{j-1})u_{2s}$. If $u_{1s}<f(y_j)/f_j$ accept y_j as a deviate. The acceptance ratio of this method is

$$\sum_{j=1}^{k} p_j[(\xi_j-\xi_{j-1})f_j]^{-1}$$

b) Let $F(y)$ be such that $f(y)=f_1(y)f_2(y)$ where the domain of y is (a, b). Let f_1 and f_2 be the maximum of $f_1(y)$ and $f_2(y)$ respectively. Then the procedure for generating random deviates having the probability density function $f(y)$ is: (1) generate U_1, U_2, U_3; (2) define $z=a+(b-a)u_3$; (3) if both $u_1<\frac{f_1(z)}{f_1}$ and $u_2<\frac{f_2(z)}{f_2}$, take z as the random deviate; otherwise take another sample of three uniform deviates. The acceptance ratio of this method is $[(b-a)f_1f_2]^{-1}$ and can be increased by dividing (a, b) into sub-intervals as in the previous case.

c) Let the probability density function of Y be

$$f(y)=\int_\alpha^\beta g(y, t)dt, \ (\alpha\leq t\leq\beta), \ (a\leq y\leq b).$$

Let g be the maximum of $g(y, t)$. Then the procedure for generating random deviates having the probability density function $f(y)$ is: (1) generate U_1, U_2, U_3; (2) define $s=\alpha+(\beta-\alpha)u_2$; $z=a+(b-a)u_3$; (3) if $u_1<\frac{g(z, s)}{g}$, take z as the random deviate; otherwise take another sample of three. The acceptance ratio for this method is $[(b-a)g]^{-1}$ and can be increased by dividing the domain of t and y into sub-domains.

5. Composition Method

Let $g_z(y)$ be a probability density function which depends on the parameter z; further let $H(z)$ be the cumulative distribution function for z. In order to generate random deviates Y having the frequency function

$$f(y)=\int_{-\infty}^{\infty} g_z(y)dH(z)$$

one draws a deviate having the cumulative distribution function $H(z)$; then draws a second sample having the probability density function $g_z(y)$.

6. Generation of Random Deviates From Well Known Distributions

a. Normal distribution

(1) *Inverse method:* The inverse method depends on having a convenient approximation to the inverse function $x=P^{-1}(u)$ where

$$u=(2\pi)^{-1/2}\int_{-\infty}^{x} e^{-t^2/2}dt.$$

Two ways of performing this operation are to (i) use **26.2.23** with $t=\left(\ln\frac{1}{u^2}\right)^{1/2}$ or (ii) approximate $x=P^{-1}(u)$ piecewise using Chebyshev polynomials, see [26.54].

(2) *Sum of uniform deviates:* Let $U_1, U_2, \ldots, U_n$ be a sequence of n uniform deviates. Then