The vertices of the triangle in the (s, t) coordinates become $A=(\sqrt{3}/4, \ -5/4)$, $B=(\sqrt{3}, \ -1)$ and $C=\left(-\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$. These points are plotted below. From the figure it is seen that the desired probability is the sum of the probabilities that the point having the transformed variables as coordinates is inside the triangles AOB, AOC, and BOC.

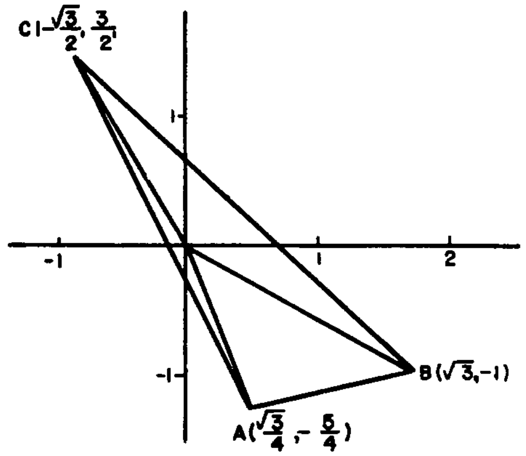

FIGURE 26.10

For these three triangles we have

	h	k_1	k_2
ΔAOB	$\frac{2}{7}\sqrt{21}$	$\sqrt{7}/14$	$\frac{4}{7}\sqrt{7}$
ΔAOC	$\frac{1}{74}\sqrt{111}$	$\frac{8}{37}\sqrt{37}$	$\frac{21}{74}\sqrt{37}$
ΔBOC	$\frac{1}{13}\sqrt{39}$	$\frac{7}{13}\sqrt{13}$	$\frac{6}{13}\sqrt{13}$

From the graph it is seen that the probability over AOB may be found in the same manner as that over **Figure 26.8**, and over AOC and BOC the probabilities may be found as that over **Figure 26.9.**

Hence

$$\iint_{\Delta} g(x, y, .5)dxdy=\iint_{\Delta ABC} g(s, t, 0)dsdt$$

$$=\iint_{\Delta AOB} g(s, t, 0)dsdt+\iint_{\Delta AOC} g(s, t, 0)dsdt$$

$$+\iint_{\Delta BOC} g(s, t, 0)dsdt$$

and consequently using **26.3.23** and **Figure 26.2**

$$\iint_{\Delta AOB} g(s, t, 0)dsdt=V\left(\frac{2}{7}\sqrt{21}, \frac{4\sqrt{7}}{7}\right)-V\left(\frac{2}{7}\sqrt{21}, \frac{\sqrt{7}}{14}\right)$$

$$=\left[\frac{1}{4}+L(1.31, 0, -.76)-L(0, 0, -.76)-\frac{1}{2}Q(1.31)\right]$$

$$-\left[\frac{1}{4}+L(1.31, 0, -.14)-L(0, 0, -.14)-\frac{1}{2}Q(1.31)\right]$$

$$=L(1.31, 0, -.76)-L(0, 0, -.76)$$

$$-L(1.31, 0, -.14)+L(0, 0, -.14)$$

$$=.00-.11-.04+.23=.08$$

$$\iint_{\Delta AOC} g(s, t, 0)dsdt=V\left(\frac{\sqrt{111}}{74}, \frac{8\sqrt{37}}{37}\right)+V\left(\frac{\sqrt{111}}{74}, \frac{21\sqrt{37}}{74}\right)$$

$$=\left[\frac{1}{4}+L(.14, 0, -.99)-L(0, 0, -.99)-\frac{1}{2}Q(.14)\right]$$

$$+\left[\frac{1}{4}+L(.14, 0, -1)-L(0, 0, -1)-\frac{1}{2}Q(.14)\right]$$

$$=.01+.02=.03$$

$$\iint_{\Delta BOC} g(s, t, 0)dsdt=V\left(\frac{\sqrt{39}}{13}, \frac{7\sqrt{13}}{13}\right)+V\left(\frac{\sqrt{39}}{13}, \frac{6\sqrt{13}}{13}\right)$$

$$=\left[\frac{1}{4}+L(.48, 0, -.97)-L(0, 0, -.97)-\frac{1}{2}Q(.48)\right]$$

$$+\left[\frac{1}{4}+L(.48, 0, -.96)-L(0, 0, -.96)-\frac{1}{2}Q(.48)\right]$$

$$=.05+.04=.09$$

Thus adding all parts, the probability that X and Y are in triangle ABC is $=.08+.03+.09=.20$. The answer to 3D is .211.

Calculation of a Circular Normal Distribution Over an Offset Circle

Example 10. Let X and Y have a circular normal distribution with $\sigma=1000$. Find the probability that the point (X, Y) falls within a circle having a radius equal to 540 whose center is displaced 1210 from the mean of the circular normal distribution.

In units of σ, the radius and displacement from the center are, respectively, $R=\frac{540}{1000}=.54$ and $r=\frac{1210}{1000}=1.21$. The problem is thus reduced to finding the probability of X and Y falling in a circle of radius $R=.54$ displaced $r=1.21$ from the center of the distribution where $\sigma=1$.